

Do now:

Solve for t

$$200 = 100 e^{0.07t}$$

Precalc Today

L63

e^{kt}

doubling time

half life

L63 questions Student pairs

7(doubling)

11/41(bacteria)

13

25&27(C-14)

$$y = Ae^{kt}$$

Finance In Exercises 7–14, complete the table for a savings account in which interest is compounded continuously.

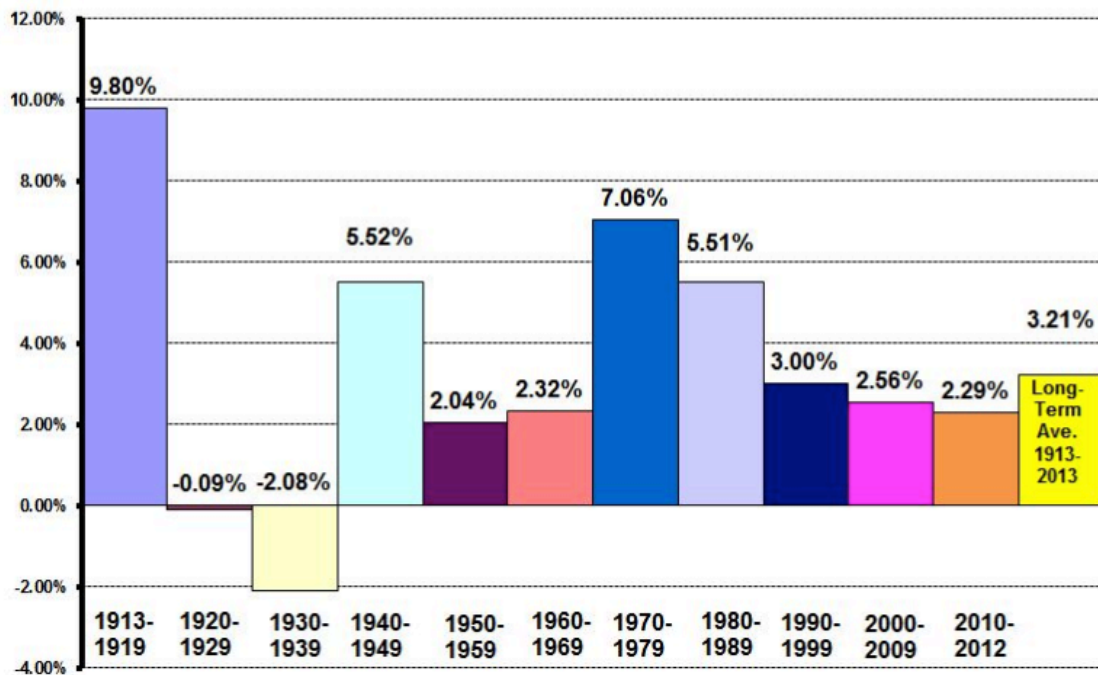
	Initial Investment A	Annual % Rate k	Time to Double T	Amount After 10 Years $y(10)$
7.	\$1000	12%		
8.	\$20,000	$10\frac{1}{2}\%$		
9.	\$750		$7\frac{3}{4}$ yr	
10.	\$10,000		12 yr	
11.	\$500			\$1505.00
12.	\$600			\$19,205.00
13.		4.5%		\$10,000.00
14.		8%		\$20,000.00

Time to Double	Amount after 10 Years	Isotope	Half-life (Years)	Initial Quantity	Amount After 1000 Years	
8 yr	\$3,320.12	25. ^{226}Ra	1620	10 g	6.52 g	41. 61.16 hr
5 yr	\$1,834.37	27. ^{14}C	5730	2.26 g	2 g	58. 4.64
yr	\$1,505.00	29. ^{239}Pu	24,360	2.16 g	2.1 g	59. 4.95
4 yr	\$10,000.00					60. $1.58 \times 10^{-}$
						61. $10^{-3.2} \approx$
						62. 10^7
						63. 10

Rule of 69 (and 70 and 72)

Average Annual Inflation by Decade

© 2013 InflationData.com
Updated 4/15/2013



RULE OF 69 (or 70 or 72)

near $\ln 2$

DOUBLING TIME IS $\frac{69}{\text{INTEREST RATE.}}$

AT $r = 10\%$ DOUBLES AFTER $\approx \frac{69}{10} \approx 6.9$ years

$69/6 \approx 11.5\%$

AT $r = 5\%$

$\approx \frac{69}{5} \approx 13.8$ years.

$69/4 \approx 17.25$ years (off by 5%)

AT $r = 3\%$

$\approx \frac{69}{3} \approx 23$ years

MATH:

$$\frac{200\%}{100\%} = \frac{100\%}{100\%} e^{kt}$$

$$2 = e^{kt}$$

$$\ln(2) = kt$$

$$0.693 = kt$$

$$t(\text{double}) = T = \frac{0.693}{k} = \frac{69.3}{\text{RATE } \ln\%}$$

Why is rule of thumb an approximation?

$$2 = (1+r)^t$$

$$\ln(2) = t \ln(1+r)$$

$$t = \frac{\ln(2)}{\ln(1+r)} \approx \frac{0.693}{\ln(1+r)}$$

$\uparrow$
approx r

5% error

$$\ln(1+0.1) \approx 0.0953$$

$$\ln(1+0.03) \approx 0.0296$$

L63

t
Time
when
amount
is 0.1g

Time
when
amount
is 0.1g

[illegible]

L52

	Isotope	Half-life (years)	Initial Quantity	Amount After 1000 Years
25.	^{226}Ra	1620	10 g	6.52 g
26.	^{226}Ra	1620	2.30 g	1.5 g
27.	^{14}C	5730	7.26 g	2 g
28.	^{14}C	5730	3 g	2.66 g
29.	^{239}Pu	24,360	2.16 g	2.1 g
30.	^{239}Pu	24,360	0.412 g	0.4 g

[illegible]

Carbon-14 Dating

C-12 is stable

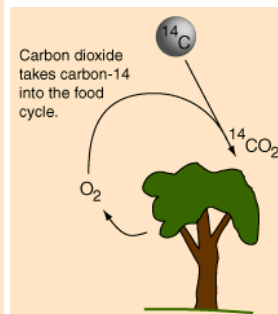
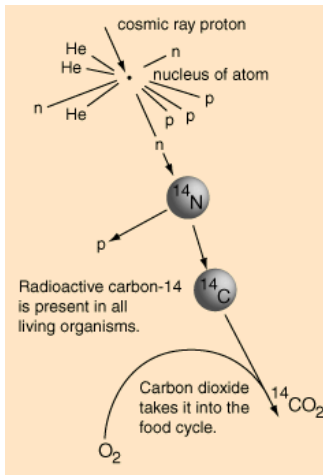
Willard Libby

near page L63

1960 Nobel Prize in Chemistry

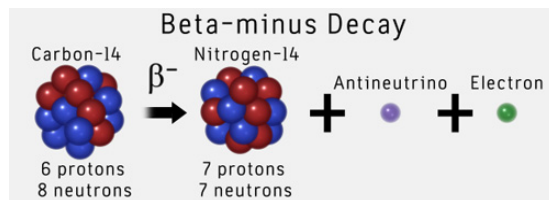
C-14 creation by cosmic rays:

2 atoms/second/square centimeter



C-14 decay rate:

15 decays/minute/gram



mass spectrometry

oxalic acid standard
versus wood from 1890

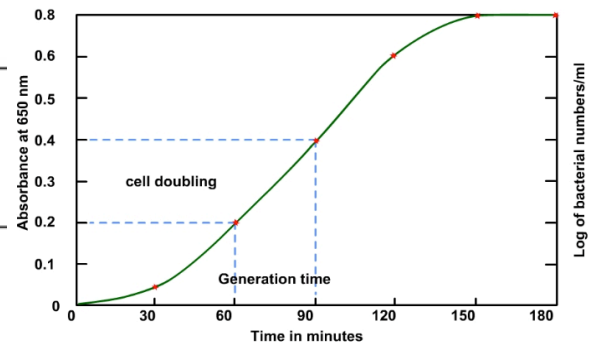
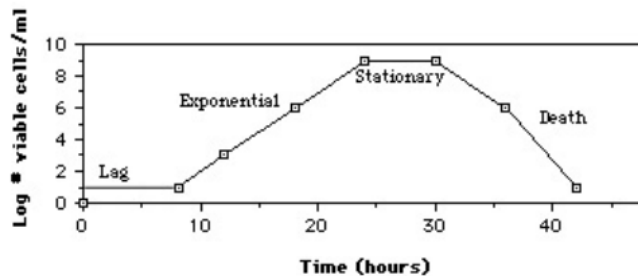
41. *Bacteria Growth* The number of bacteria N in a culture is modeled by

$$N = 250e^{kt}$$

where t is the time in hours. If $N = 280$ when $t = 10$, estimate the time required for the population to double in size.

Bacterial Growth

Near pages L63-L64



Doubling times (Generation times) for some common bacteria under optimal conditions of growth.

Bacterium	Medium	Generation Time (minutes)
• <i>Escherichia coli</i>	Glucose-salts	17
<i>Bacillus megaterium</i>	Sucrose-salts	25
<i>Streptococcus lactis</i>	Milk	26
<i>Streptococcus lactis</i>	Lactose broth	48
<i>Staphylococcus aureus</i>	Heart infusion broth	27-30
• <i>Lactobacillus acidophilus</i>	Milk	66-87
<i>Rhizobium japonicum</i>	Mannitol-salts-yeast extract	344-461
<i>Mycobacterium tuberculosis</i>	Synthetic	792-932
<i>Treponema pallidum</i>	Rabbit testes	1980

1. Solve for the unknowns to 3 significant figures.
Use algebra. You may check with the calculator.

(a) $30 = 60e^{-0.05t}$

(b) $120 = Ae^{0.7}$

2. The number of bacteria in a culture, y , is modeled by $y = 400e^{kt}$, where t is the time in hours.

(a) Find k if $(t, y) = (3, 600)$ is on the graph of y . Write k to six decimals.

(b) Using part (a), find the number of bacteria y after $t = 4$ hours. Round down.

(c) Find the doubling time, T , to 4 decimals.

(d) Find the value of t for which $y = 1000$ bacteria.

3. The radioactive isotope Rubikskubium-299, ^{299}Rk , could have a half life of 3 attoseconds. If there were 40 femtograms of the isotope at zero attoseconds, how much of the isotope is there after the following time intervals?
- (a) 12 attoseconds (answer exactly).
- (b) 14 attoseconds (answer to two decimals).
4. A beetle population is growing exponentially. If the continuous exponential growth *rate constant* is 20% per day and there are 1,000 beetles now, how many beetles will there be after 14 days? Round down to the nearest beetle.

5. A student began taking the ACT at 8am ($t = 0$ hours). The student had 100% brain energy (y in percent) at that time. By 12:30pm ($t = 4.5$ hours), the student had 30% brain energy left. Use a continuous exponential decay model to find the following.

(a) Find the half life (T in hours) of brain energy.
Answer to the nearest 10th of an hour.

(b) Predict the brain energy that was left at 10am.
Answer to the nearest percent.

1. Solve for the unknowns to 3 significant figures.
Use algebra. You may check with the calculator.

$$(a) \quad \frac{30}{60} = \frac{60}{60} e^{-0.05t} \quad \left| \quad \frac{\ln(\frac{1}{2})}{-0.05} = \frac{-0.05t}{-0.05} \right.$$

$$\frac{1}{2} = e^{-0.05t} \quad \left| \quad \frac{\ln(0.5)}{-0.05} = t = 13.8 \right.$$

$$(b) \quad \frac{120}{e^{0.7}} = \frac{Ae^{0.7}}{e^{0.7}} \quad \left| \quad A = 120/e^{0.7} \right.$$

$e^{0.7}$ is just a number.

$$= 59.6$$

2. The number of bacteria in a culture, y , is modeled by $y = 400e^{kt}$, where t is the time in hours.

- (a) Find k if $(t, y) = (3, 600)$ is on the graph of y .
Write k to six decimals.

Solve for k because the formula is complete.

$$600 = 400e^{k \cdot 3}$$

$$\ln\left(\frac{600}{400}\right) = 3k \quad \left| \quad 3k = \ln(1.5) \right.$$

$$k = \ln(1.5)/3 = 0.135155/\text{hr.}$$

t	y
0	400
3	600
4	686
5.1285	800
6.7796	1000

- (b) Using part (a), find the number of bacteria y after $t = 4$ hours. Round down.

$$y = 400e^{0.135155 \cdot 4}$$

$$y = 400e^{0.54062} = 686.8 = 686$$

- (c) Find the doubling time, T , to 4 decimals.

$$\frac{800}{400} = \frac{400e^{0.135155T}}{400}$$

DOUBLE START

$$2 = e^{0.135155T}$$

$$\ln(2) = 0.135155T$$

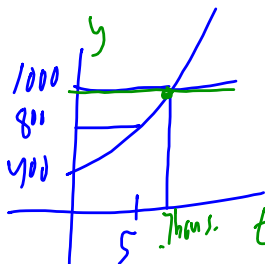
$$T = \frac{\ln(2)}{0.135155} = 5.1285 \text{ hrs.}$$

- (d) Find the value of t for which $y = 1000$ bacteria.

$$1000 = 400e^{0.135155t}$$

$$\ln(2.5) = 0.135155t$$

$$t = 6.7796 \text{ hours}$$



3. The radioactive isotope Rubikskubium-299, ^{299}Rk , could have a half life of 3 attoseconds. If there were 40 femtograms of the isotope at zero attoseconds, how much of the isotope is there after the following time intervals?

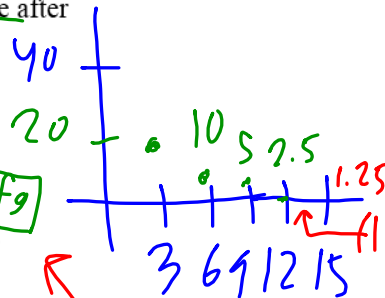
(a) 12 attoseconds (answer exactly).

Divide by 2 4 times: $\frac{40}{2^4} = 2.5 \text{ fg}$

(b) 14 attoseconds (answer to two decimals).

$$y = A\left(\frac{1}{2}\right)^{t/p}$$

$$y = 40\left(\frac{1}{2}\right)^{t/3} = 40\left(\frac{1}{2}\right)^{14/3} = 1.57 \text{ fg}$$



Formula method

$$y = 40\left(\frac{1}{2}\right)^{12/3}$$

$$= 40\left(\frac{1}{2}\right)^4$$

$$= 40\left(\frac{1}{2^4}\right) = \frac{40}{16} = 2.5$$

4. A beetle population is growing exponentially. If the continuous exponential growth rate constant is 20% per day and there are 1,000 beetles now, how many beetles will there be after 14 days? Round down to the nearest beetle.

$$k = 20\% / \text{day} = 0.2 / \text{day}$$

$$y = Ae^{kt}$$

$$y = 1000e^{0.2 \cdot 14} = 1000e^{(0.2 \cdot 14)} = 16444.6 = 16,444 \text{ beetles}$$

5. A student began taking the ACT at 8am ($t = 0$ hours). The student had 100% brain energy (y in percent) at that time. By 12:30pm ($t = 4.5$ hours), the student had 30% brain energy left. Use a continuous exponential decay model to find the following.



- (a) Find the half life (T in hours) of brain energy.
Answer to the nearest 10th of an hour.

$$y = A \left(\frac{1}{2} \right)^{t/T}$$

$$\frac{30}{100} = \frac{100}{100} \left(\frac{1}{2} \right)^{4.5/T}$$

$$\ln(0.3) = \ln \left(\left(\frac{1}{2} \right)^{4.5/T} \right)$$

$$y = A e^{kt}$$

t	y
0	100%
2.6	59%
4.5	30%

$T = 2.6h$

59%

- (b) Predict the brain energy that was left at 10am.
Answer to the nearest percent.

$$\ln(0.3) = \frac{4.5}{T} \ln \left(\frac{1}{2} \right)$$

$$\text{Solve for } T. \quad T = \frac{4.5 \ln(1/2)}{\ln(0.3)} = 2.6h$$

$$y = 100 \left(\frac{1}{2} \right)^{2/2.6} = 59\%$$

Exponential Models 4 Practice

Name:

A colony of bacteria grow in number exponentially according to $y = Ae^{kt}$.

Given: 100 bacteria initially.

Given: 300 bacteria after 20 hours.

It may help to make a table of (t, y) .

a) Find k to 5 decimals. What are the units?

b) Find the number of bacteria after 10 hours. Why is it not 200 bacteria (midway between 100 and 300)?

c) Find the number of bacteria after 30 hours. (It's not 400.)

d) Find the doubling time. Guess first.

e) After how long will there be 800 bacteria? Guess first.

Exponential Models 4 Practice

Name: Student
lead
ans.

A colony of bacteria grow in number exponentially according to $y = Ae^{kt}$.

Given: 100 bacteria initially. A

Given: 300 bacteria after 20 hours.

It may help to make a table of (t, y) .

$$y = Ae^{kt}$$

a) Find k to 5 decimals. What are the units?

$$300 = 100e^{k(20)}$$
$$\frac{\ln(3)}{20} = \frac{20k}{20} \quad k = .05493$$

b) Find the number of bacteria after 10 hours. Why is it not 200 bacteria (midway between 100 and 300)?

$$y = 100e^{.05493(10)} = 173$$

c) Find the number of bacteria after 30 hours. (It's not 400.)

$$y = 100e^{.05493(30)} = 519$$

d) Find the doubling time. Guess first.

$$200 = 100e^{.05493t}$$

$$(12.62 \text{ hours})$$

e) After how long will there be 800 bacteria? Guess first.

$$800 = 100e^{.05493t}$$

$$(37.856 \text{ hr})$$