

Do Now:

Given the arithmetic sequence

$$4, -1, -6, \dots$$

Find the 22nd term and the sum of the first 22 terms.

Step 1. Find a_n $d = -5$

$$a_n = 4 + (n-1)(-5)$$
$$4 + (21)(-5)$$
$$4 - 105 = -101$$

Step 2. Find S_n

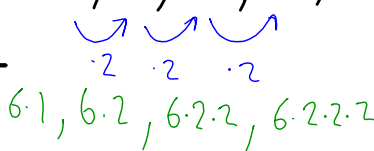
$$\left(\frac{4 + (-101)}{2} \right) 22 = \frac{-97}{2} 22 = 11 \cdot 97 = -(970 + 9) = -1067$$

Geometric Sequences

$$\frac{12}{6} = 2 \quad \frac{24}{12} = 2 \quad \frac{48}{24} = 2 \quad \dots \quad \frac{a_n}{a_{n-1}} = 2 = \boxed{r}$$

Ex. Find formula: 6, 12, 24, 48, ...

HAS A common RATIO.



formula $a_n = 6 \cdot (2)^{n-1}$

Write the formula

a) 1, 3, 9, 27, 81, 243, 729, 2187

$\cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

$a_n = 1 \cdot 3^{n-1}$

$$a_n = a_1 (r)^{n-1}$$

FORMULA FOR ANY GEOMETRIC SEQUENCE.

b) $\frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}, \frac{1}{256}, \frac{1}{512}, \frac{1}{1024}, \frac{1}{2048}, \frac{1}{4096}, \frac{1}{8192}, \frac{1}{16384}$

$\cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

$a_n = \frac{1}{8} \left(\frac{1}{2}\right)^{n-1}$

$= \frac{1}{8} \left(\frac{1}{2}\right)^n \left(\frac{1}{2}\right)^{-1} = \frac{1}{8} \left(\frac{1}{2}\right)^{n-1} = \frac{1}{4} \left(\frac{1}{2}\right)^n$

c) 1, -1, 1, -1, 1, ... $r = -1$

$a_n = 1(-1)^{n-1} = (-1)^n (-1)^{-1} = (-1)^n (-1) = -(-1)^n$

$\frac{1}{-1} = -1$

d) $\frac{1}{2}, -\frac{1}{8}, \frac{1}{32}, \dots, \frac{1}{128}, \frac{1}{512}, \dots$

$\cdot -\frac{1}{4} \cdot -\frac{1}{4} \cdot -\frac{1}{4}$

$a_n = \frac{1}{2} \left(-\frac{1}{4}\right)^{n-1} = \frac{1}{2} \left(-\frac{1}{4}\right)^n \left(-\frac{1}{4}\right)^{-1}$

OK $\frac{1}{2} \left(-\frac{1}{4}\right)^n (-4)$

$-2 \left(-\frac{1}{4}\right)^n$

Geometric Series

Sum of the terms of a geometric sequence.

Given: 4, 8, 16, 32. (Finite, has a last term)
 $r=2$

The sum is

$$S_4 = 4 + 8 + 16 + 32 = 60$$

$$1. S_4 = 4 \cdot 2^0 + 4 \cdot 2^1 + 4 \cdot 2^2 + 4 \cdot 2^3$$

$$2. S_4 = 4 \cdot 2^1 + 4 \cdot 2^2 + 4 \cdot 2^3 + 4 \cdot 2^4$$

$$2S_4 - S_4 = 4 \cdot 2^4 - 4 \cdot 2^0$$

$$S_4 = 4 \frac{2^4 - 2^0}{2 - 1} = 4 \frac{16 - 1}{2 - 1} = 4 \frac{15}{1} = \boxed{60} \text{ (i)}$$

$$S_n = a_1 \left(\frac{1 - r^n}{1 - r} \right)$$

$$\left. \begin{aligned} a_n &= a_1 + (n-1)d \\ S_n &= \frac{1}{2}(a_1 + a_n) \cdot n \end{aligned} \right\} \text{ARITHMETIC SEQUENCES}$$

$$\left. \begin{aligned} a_n &= a_1(r)^{n-1} \\ S_n &= a_1 \left(\frac{1-r^n}{1-r} \right) \end{aligned} \right\} \text{GEOMETRIC SEQUENCES} \\ \text{(Finite)}$$

→ SAME AS $S_n = a_1 \left(\frac{r^n - 1}{r - 1} \right)$.

For $4 + 8 + 16 + 32$

Find

$$S_4 = 4 \left(\frac{1-2^4}{1-2} \right) = 4 \left(\frac{1-16}{-1} \right) = 4 \left(\frac{-15}{-1} \right) = 4 \left(\frac{15}{1} \right) = \boxed{60}$$

For $6 + 18 + 54 + \dots$ $r=3$

Find

$$S_7 = 6 \left(\frac{1-3^7}{1-3} \right) = 6 \left(\frac{1-2187}{1-3} \right) = 6 \left(\frac{-2186}{-2} \right) = 6558$$

Given: $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots$

Find

S_{10}

Find $1 - 1 + 1 - 1 + 1 - 1 + 1 \dots$