Do Now:

Given the arithmetic sequence

$4, -1, -6, \ldots$

Find the 22nd term and the sum of the first 22 terms.

Step 1. Find a_n

$a_n = 4 + (n-1)(-5)$ $\quad d = -5$

$4 + (21)(-5)$

$4 - 105 = -101$

Step 2. Find S_n

$$\left(\frac{4 + -101}{2}\right)22 = \frac{-97}{2}22 = 11 \cdot 97 = -(970 + 97) = -1067$$

Geometric Sequences

$\frac{12}{6}=2 \quad \frac{24}{12}=2 \quad \frac{48}{24}=2 \quad \cdots \frac{a_n}{a_{n-1}}=2 \boxed{=r}$

Ex. Find formula: $6, 12, 24, 48, \ldots$

HAS A COMMON RATIO.

$\cdot 2 \quad \cdot 2 \quad \cdot 2$

$6\cdot 1,\ 6\cdot 2,\ 6\cdot 2\cdot 2,\ 6\cdot 2\cdot 2\cdot 2$

formula

$a_n = 6\cdot(2)^{n-1}$

Write the formula

$\boxed{a_n = a_1(r)^{n-1}}$

FORMULA FOR ANY GEOMETRIC SEQUENCE.

a) $1, 3, 9, 27, 81, \ldots, 243, 729, 2187$

$\cdot 3 \quad \cdot 3 \quad \cdot 3 \quad \cdot 3 \quad \cdot 3 \quad \cdot 3 \quad \cdot 3$

$a_n = 1\cdot 3^{n-1}$

b) $\frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}, \frac{1}{256}, \frac{1}{512}, \frac{1}{1024}, \frac{1}{2048}, \frac{1}{4096}, \frac{1}{8192}$

$\cdot\frac{1}{2} \quad \cdot\frac{1}{2} \quad \cdot\frac{1}{2}$

$\frac{1}{4}$

$a_n = \frac{1}{8}\left(\frac{1}{2}\right)^{n-1}$

$= \frac{1}{8}\left(\frac{1}{2}\right)^n\left(\frac{1}{2}\right)^{-1} = \frac{1}{8}\left(\frac{1}{2}\right)^n\left(\frac{2}{1}\right) = \frac{1}{4}\left(\frac{1}{2}\right)^n$

c) $1, -1, 1, -1, 1, \ldots \quad r=-1$

$a_n = 1(-1)^{n-1} = (-1)^n(-1)^{-1} = (-1)^n(-1) = \boxed{-(-1)^n}$

$\frac{1}{-1} = -1$

d) $\frac{1}{2}, -\frac{1}{8}, \frac{1}{32}, \ldots, \frac{1}{128}, \frac{1}{512}, \ldots$

$\cdot -\frac{1}{4} \quad \cdot -\frac{1}{4} \quad \cdot -\frac{1}{4}$

$\underbrace{a_n = \frac{1}{2}\left(-\frac{1}{4}\right)^{n-1}}_{\text{OK}} = \frac{1}{2}\left(-\frac{1}{4}\right)^n\left(-\frac{1}{4}\right)^{-1}$

$\frac{1}{2}\left(-\frac{1}{4}\right)^n(-4)$

$\boxed{-2\left(-\frac{1}{4}\right)^n}$

Geometric Series

Sum of the terms of a geometric sequence.

Given: 4, 8, 16, 32. (Finite, has a last term)

$r = 2$

$4(2)^{n-1}$

The sum is

$S_4 = 4 + 8 + 16 + 32 = 60$

$r \cdot S_4$

$S_4 = 4 \cdot 2^0 + \cancel{4 \cdot 2^1} + \cancel{4 \cdot 2^2} + \cancel{4 \cdot 2^3}$.

$2 \cdot S_4 = \cancel{4 \cdot 2^1} + \cancel{4 \cdot 2^2} + \cancel{4 \cdot 2^3} + 4 \cdot 2^4$.

$2S_4 - S_4 = 4 \cdot 2^4 - 4 \cdot 2^0$

$S_4 = 4\,\dfrac{2^4 - 2^0}{2-1} = 4\,\dfrac{16-1}{2-1} = 4\,\dfrac{15}{1} = \boxed{60}$ ☺

$$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right)$$

$$a_n = a_1 + (n-1)d$$
$$S_n = \frac{1}{2}(a_1 + a_n)\cdot d$$

ARITHMETIC SEQUENCES

$$a_n = a_1 (r)^{n-1}$$
$$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$$

GEOMETRIC SEQUENCES (Finite)

SAME AS $S_n = a_1\left(\frac{r^n-1}{r-1}\right)$.

For $4+8+16+32$

Find

$$S_4 = 4\left(\frac{1-2^4}{1-2}\right) = 4\left(\frac{1-16}{-1}\right) = 4\left(\frac{-15}{-1}\right) = 4\left(\frac{15}{1}\right) = \boxed{60}.$$

For $6+18+54+\ldots$ $r=3$

Find

$$S_7 = 6\left(\frac{1-3^7}{1-3}\right) = 6\left(\frac{1-2187}{1-3}\right) = 6\left(\frac{-2186}{-2}\right) = 6558$$

Given: $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots$

Find

S_{10}

Find $1-1+1-1+1-1+1\ \ldots$