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## For Practice

1. In the 2008 NFL regular season, quarterback Kurt Warner of the Arizona Cardinals had an average of 286 passing yards with a standard deviation of 103 yards.
  - (a) Calculate and interpret the  $z$ -score for a game in which he threw for 361 yards.
  - (b) Calculate and interpret the  $z$ -score for a game in which he threw for 192 yards.
  - (c) If the  $z$ -score for a particular game was 1.30, how many yards did he throw for?
2. In the 2008–2009 NBA regular season, LeBron James of the Cleveland Cavaliers averaged 28.4 points per game with a standard deviation of 8.8 points per game.
  - (a) Calculate and interpret the  $z$ -score for a game in which he scored 40 points.
  - (b) Calculate and interpret the  $z$ -score for a game in which he scored 25 points.
  - (c) If the  $z$ -score for a particular game was  $-1.86$ , how many points did he score?
3. Knowing that a *PERFORMANCE* is above average doesn't tell us how great it is. Describe a situation in which being 10 above average is an unusually great *PERFORMANCE* and a situation in which being 10 above average is a good *PERFORMANCE*, but not great.
4. Knowing that a *PERFORMANCE* is below average doesn't tell us how poor it is. Describe a situation in which being 5 below average is an unusually poor *PERFORMANCE* and a situation in which being 5 below average is a poor *PERFORMANCE*, but not unusually poor.
5. In the 2008 baseball season, first baseman Ryan Howard of the Philadelphia Phillies led the Major Leagues with 48 home runs (HR) and 146 runs batted in (RBI). Overall, using all players with at least 300 plate appearances, the mean number of HR was 14.9 with a standard deviation of 9.8, whereas the mean number of RBI was 62.5 with a standard deviation of 25.5. Which was the more impressive *PERFORMANCE*? Explain, using the methods of this chapter.
6. Of the 281 players with at least 300 plate appearances in the 2008 Major League Baseball season, Jeff Mathis of the Los Angeles Angels had the lowest batting average (0.194) and Brad Wilkerson of the Toronto Blue Jays scored the fewest runs (21). Overall, the mean batting average was 0.272, with a standard deviation of 0.027, and the mean number of runs scored was 65.0, with a standard deviation of 23.6. Which *PERFORMANCE* was worse? Explain, using the methods of this chapter.

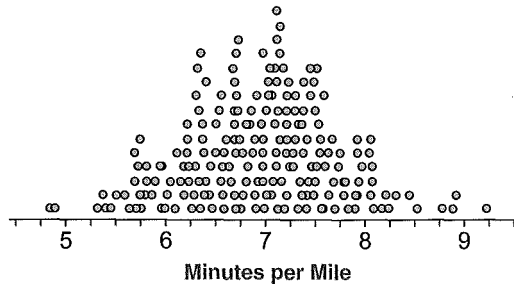
7. In 2004, Ichiro Suzuki broke the Major League Baseball record for hits in a season (262 hits) that had been held by George Sisler since 1920 (257 hits). The mean number of hits for everyday players in 1920 was 158 with a standard deviation of 30. The mean number of hits for everyday players in 2004 was 155 with a standard deviation of 25. Who had the better *PERFORMANCE*, Sisler or Ichiro? Explain your reasoning.
8. In 2008, Drew Brees of the New Orleans Saints nearly broke Dan Marino's record, set in 1984, for most passing yards in a season. Brees has 5069 yards, just shy of Marino's 5084. Using the top 28 quarterbacks of 1984 (since there were 28 teams), the mean number of yards was 2839 with a standard deviation of 927. The mean number of yards for the top 32 quarterbacks in 2008 was 3132 with a standard deviation of 886. Who had the better *PERFORMANCE*, Marino or Brees? Explain your reasoning.
9. Suppose that you and your friends decided to start a fantasy basketball league using NBA players. You will keep track of your players' statistics for 4 variables: points, rebounds, assists, and turnovers, and each of these variables will count equally in the overall standings. The table shows the mean and standard deviation for each of these variables using data from the 2008–2009 regular season and only considering players with at least 60 games played. The table also shows the *PERFORMANCES* of three individual players. Using the methods of this chapter, rank these players in order of overall fantasy value. *Hint:* Since being above average in turnovers is bad, add the z-scores for points, rebounds, and assists and subtract the z-score for turnovers to measure overall fantasy value.

|                           | POINTS      | REBOUNDS   | ASSISTS    | TURNOVERS  |
|---------------------------|-------------|------------|------------|------------|
| Dwayne Wade               | 30.2        | 5.0        | 7.5        | 3.4        |
| Dirk Nowitzki             | 25.9        | 8.4        | 2.4        | 1.9        |
| Dwight Howard             | 20.6        | 13.8       | 1.4        | 3.0        |
| <b>Mean</b>               | <b>11.1</b> | <b>4.5</b> | <b>2.4</b> | <b>1.5</b> |
| <b>Standard deviation</b> | <b>5.8</b>  | <b>2.4</b> | <b>2.0</b> | <b>0.7</b> |

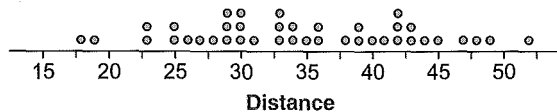
10. Suppose that you and your friends decided to start a fantasy baseball league using Major League Baseball players. You will keep track of your pitchers' statistics for 3 variables: wins, strikeouts, and ERA (average numbers of runs given up per 9 innings), and each of these variables will count equally in the overall standings. The table shows the mean and standard deviation for each of these variables using data from the 2009 regular season and only considering pitchers with at least 100 innings pitched. The table also shows the *PERFORMANCES* of three individual pitchers. Using the methods of this chapter, rank these players in order of overall fantasy value. *Hint:* Since being above average in ERA is bad, add the z-scores for wins and strikeouts and subtract the z-score for ERA to measure overall fantasy value.

|                           | WINS        | STRIKEOUTS   | ERA         |
|---------------------------|-------------|--------------|-------------|
| Tim Lincecum              | 15          | 261          | 2.48        |
| Chris Carpenter           | 17          | 144          | 2.24        |
| Adam Wainwright           | 19          | 212          | 2.63        |
| <b>Mean</b>               | <b>10.6</b> | <b>128.4</b> | <b>4.28</b> |
| <b>Standard deviation</b> | <b>3.8</b>  | <b>48.4</b>  | <b>0.95</b> |

- In the Olympics, participants in the Decathlon compete in 10 different track and field events. In each event, participants are awarded up to 1000 points for each *PERFORMANCE*; the winner is the decathlete with the highest point total. Explain how you could use standardized scores to determine a winner rather than using points.
- Suppose that you wanted to have a scholar-athlete competition at your school in which each participant competes in 5 academic and 5 athletic contests. Describe which contests you would include and how you would determine a winner using standardized scores.
- A distance runner recorded her average time (in minutes) per mile during each of her 200 training runs over the last year. Using the dotplot below, estimate the standard deviation of her distribution of minutes per mile. Explain your reasoning.



- A punter on a high school football team recorded the distance (in yards) that each of his punts traveled during the last season. Using the dotplot below, estimate the standard deviation of his distribution of distances. Explain your reasoning.



- During his football career, running back Emmitt Smith earned a place as one of the best rushers of all time. Over the 226 games in his career, his rushing

yards per game average was 81.2 yards/game with a standard deviation of 42.7 yards/game. Also, the distribution of these 226 values is roughly symmetric, unimodal, and bell-shaped.

- (a) Sketch what this distribution should look like by drawing a bell-shaped curve and labeling the mean, mean  $\pm 1$  SD, and mean  $\pm 2$  SD.
  - (b) Between which two values would you expect the middle 95% of his *PERFORMANCES* to be?
  - (c) What percent of his *PERFORMANCES* would you expect to be above 123.9 yards? Explain.
  - (d) Calculate and interpret the *z*-score for his best game: 237 yards. Would you consider this an exceptional *PERFORMANCE*? Explain.
16. During the 2008–2009 regular season, basketball player Kobe Bryant of the Los Angeles Lakers averaged 26.8 points per game with a standard deviation of 8.6 points per game. Furthermore, the distribution of his points is roughly symmetric, unimodal, and bell-shaped.
- (a) Sketch what this distribution should look like by drawing a bell-shaped curve and labeling the mean, mean  $\pm 1$  SD, and mean  $\pm 2$  SD.
  - (b) Between which two values would you expect the middle 68% of his *PERFORMANCES* to be?
  - (c) What percent of his *PERFORMANCES* would you expect to be below 9.6 points? Explain.
  - (d) Calculate and interpret the *z*-score for his best game: 61 points. Would you consider this an exceptional *PERFORMANCE*? Explain.
17. After years of practicing at the local bowling alley, Allan has determined that his distribution of bowling scores is roughly symmetric, unimodal, and bell-shaped, with a mean of 182 points and a standard deviation of 23 points.
- (a) Sketch what this distribution should look like by drawing a bell-shaped curve and labeling the mean, mean  $\pm 1$  SD, and mean  $\pm 2$  SD.
  - (b) Between which two values would you expect the middle 68% of his *PERFORMANCES* to be?
  - (c) How likely is it that Allan will roll a perfect game (300 points), just by *RANDOM CHANCE*? Explain.
18. During her senior year, Keri has determined that her distribution of discus distances is roughly symmetric, unimodal, and bell-shaped, with a mean of 113 feet and a standard deviation of 3.1 feet.
- (a) Sketch what this distribution should look like by drawing a bell-shaped curve and labeling the mean, mean  $\pm 1$  SD, and mean  $\pm 2$  SD.
  - (b) Between which two values would you expect the middle 95% of her *PERFORMANCES* to be?
  - (c) Is it likely that Keri will achieve her personal goal for the season (120 feet)? Explain.

19. In 2004, the distribution of triples for Major League Baseball players with a minimum of 500 plate appearances had a mean of 3.12 triples, with a standard deviation of 3.16 triples. Sketch what this distribution would look like *if* the distribution was roughly symmetric, unimodal, and bell-shaped by drawing a bell-shaped curve and labeling the mean, mean  $\pm 1$  SD, and mean  $\pm 2$  SD. Based on your sketch, is it plausible that the distribution of triples is unimodal and symmetric? Explain.
20. In 2004, the distribution of runs scored for Major League Baseball players with a minimum of 500 plate appearances had a mean of 84.5 runs, with a standard deviation of 18.6 runs. Sketch what this distribution would look like *if* the distribution was roughly symmetric, unimodal, and bell-shaped by drawing a bell-shaped curve and labeling the mean, mean  $\pm 1$  SD, and mean  $\pm 2$  SD. Based on your sketch, is it plausible that the distribution of runs scored is unimodal and symmetric? Explain.
21. In the 2008 Wimbledon tennis tournament, Rafael Nadal averaged 115 miles per hour (mph) on his first-serve.<sup>3</sup> Assuming that the distribution of his first-serve speeds can be modeled by a Normal distribution with a standard deviation of 6 mph, use z-scores and the Normal table to answer the following questions:
- (a) About what proportion of Nadal's first-serve would you expect to exceed 118 mph?
  - (b) About what proportion of his first-serve would you expect to be less than 125 mph?
22. On the 2009 PGA tour, Tiger Woods had an average driving distance of 298 yards.<sup>4</sup> Assuming that the distribution of his driving distances can be modeled by a Normal distribution with a standard deviation of 12 yards, use z-scores and the Normal table to answer the following questions:
- (a) About what proportion of his drives would you expect to be less than 270 yards?
  - (b) About what proportion of his drives would you expect to be more than 280 yards?
23. In the 2008 Wimbledon tennis tournament, Rafael Nadal averaged 115 miles per hour (mph) on his first-serve. Assuming that the distribution of his first-serve speeds can be modeled by a Normal distribution with a standard deviation of 6 mph, use z-scores and the Normal table to answer the following questions:
- (a) About what proportion of his first-serve would you expect to be between 110 and 120 mph?
  - (b) In a particular match, Nadal had 50 first-serve. About how many of the first-serve do you expect will be between 110 and 120 mph?
24. On the 2009 PGA tour, Tiger Woods had an average driving distance of 298 yards. Assuming that the distribution of his driving distances can be modeled by a Normal distribution with a standard deviation of 12 yards, use z-scores and the Normal table to answer the following questions:
- (a) About what proportion of his drives would you expect to be between 290 and 310 yards?

- (b) In a particular round, Tiger plans to use his driver 14 times. About how many of his drives do you expect will go between 290 and 310 yards?
25. Suppose that Rafael Nadal was able to make the speed of his first-serve more consistent, so that the standard deviation of his first-serve speeds is smaller than 6 mph. How will this affect the answers to the questions in Exercise 23? *Hint:* Sketch two distributions and label the mean, mean  $\pm 1$  SD, and mean  $\pm 2$  SD.
26. Suppose that Tiger Woods's driving distances have become less consistent, so that the standard deviation of his driving distance is greater than 12 yards. How will this affect the answers to the questions in Exercise 24? *Hint:* Sketch two distributions and label the mean, mean  $\pm 1$  SD, and mean  $\pm 2$  SD.
27. Suppose that a basketball player's scoring average places her at the 80th percentile of all players in her league. Explain what it means to be at the 80th percentile.
28. To qualify for an elite track meet, a long-jumper must be in the top 5% of all jumpers in his state. To qualify for the meet, what percentile must the jumper exceed? Explain.
29. In the 2008 Wimbledon tennis tournament, Rafael Nadal averaged 115 miles per hour (mph) on his first-serve. Assuming that the distribution of his first-serve speeds can be modeled by a Normal distribution with a standard deviation of 6 mph, use z-scores and the Normal table to answer the following questions:
- (a) The fastest 15% of Nadal's first-serve are above what speed?
  - (b) What is the 25th percentile of his distribution of first-serve speeds? What is another name for this boundary?
30. On the 2009 PGA tour, Tiger Woods had an average driving distance of 298 yards. Assuming that the distribution of his driving distances can be modeled by a Normal distribution with a standard deviation of 12 yards, use z-scores and the Normal table to answer the following questions:
- (a) The farthest 25% of Tiger's drives are above what distance? What is another name for this boundary?
  - (b) What is the 5th percentile of his distribution of driving distances?
31. Suppose that pitcher Zach Greinke throws his fastball with an average speed of 94 miles per hour (mph) and a standard deviation of 2 mph, and that the distribution of his fastball speeds can be modeled by a Normal distribution.<sup>5</sup>
- (a) About what proportion of his fastballs will travel over 100 mph?
  - (b) About what proportion of his fastballs will travel less than 90 mph?
  - (c) About what proportion of his fastballs will travel between 93 and 95 mph?
  - (d) What is the 30th percentile of Greinke's distribution of fastball speeds?

32. Suppose golfer Michelle Wie has an average driving distance of 269.1 yards,<sup>6</sup> with a standard deviation of 13.8 yards, and that the distribution of her driving distances can be modeled by a Normal distribution.
- (a) About what proportion of her drives will go more than 260 yards?
  - (b) About what proportion of her drives will go less than 285 yards?
  - (c) About what proportion of her drives will go between 265 and 275 yards?
  - (d) The longest 10% of her drives will go farther than what distance?
33. In Chapter 4, we learned the  $1.5IQR$  rule to identify unusually low or high *PERFORMANCES*. Specifically, we called a *PERFORMANCE* an outlier if it was less than  $Q_1 - 1.5IQR$  or greater than  $Q_3 + 1.5IQR$ . Referring to the information in Exercise 31, what fastball velocities would be considered low outliers for Zach Greinke?
34. In Chapter 4, we learned the  $1.5IQR$  rule to identify unusually low or high *PERFORMANCES*. Specifically, we called a *PERFORMANCE* an outlier if it was less than  $Q_1 - 1.5IQR$  or greater than  $Q_3 + 1.5IQR$ . Referring to the information in Exercise 32, what driving distances would be considered high outliers for Michelle Wie?
35. Suppose that a baseball pitcher's fastballs have an average speed of 92 mph and 40% of his fastballs go less than 90 mph. What is the standard deviation of his fastball speeds, assuming the distribution of his fastball speeds can be modeled by a Normal distribution?
36. Suppose that a golfer has an average driving distance of 265 yards and 15% of her drives go less than 250 yards. What is her standard deviation of driving distances, assuming the distribution of her driving distances can be modeled by a Normal distribution?
37. A basketball player with the *ABILITY* to make 70% of his free-throws takes 150 free-throws during the first month of the season. However, because of *RANDOM CHANCE*, his *PERFORMANCE* when taking 150 free-throws is unlikely to be exactly 70%. In fact, his free-throw shooting percentage for sets of 150 free-throws can be modeled by a Normal distribution with a mean of 70% and a standard deviation of 3.7%.
- (a) Sketch what this distribution should look like by drawing a Normal curve and labeling the mean, mean  $\pm 1$  SD, and mean  $\pm 2$  SD.
  - (b) During the first month of the season, he makes only 65% of his free-throws. Assuming his *ABILITY* is still 70%, is he likely to have a better shooting percentage in his next 150 free-throws? Justify your answer.
38. A tennis player with the *ABILITY* to make 75% of her first-serve attempts 50 first-serve attempts during the first match of a tournament. However, because of *RANDOM CHANCE*, her *PERFORMANCE* when attempting 50 first-serve attempts is unlikely to be exactly 75%. In fact, her first-serve percentage for sets of 50 first-serve attempts can be modeled by a Normal distribution with a mean of 75% and a standard deviation of 6.1%.

- (a) Sketch what this distribution should look like by drawing a Normal curve and labeling the mean, mean  $\pm 1$  SD, and mean  $\pm 2$  SD.
- (b) During the first match of the tournament, she makes 86% of her first-serves. Is she likely to have another *PERFORMANCE* of at least 86% during the second match of the tournament, assuming she takes 50 first-serves and her *ABILITY* is still 75%? Justify your answer.

## CHAPTER REVIEW EXERCISES

39. In the 1994–1995 NBA regular season, David Robinson averaged 27.6 points per game with a standard deviation of 7.3 points, and the distribution of his *PERFORMANCES* can be modeled by a Normal distribution.
- (a) Sketch what this distribution should look like by drawing a Normal curve and labeling the mean, mean  $\pm 1$  SD, and mean  $\pm 2$  SD.
  - (b) Calculate and interpret the z-score for the game where he scored only 11 points.
  - (c) Without doing any additional calculations, would you describe a *PERFORMANCE* of 11 points as unusually low? Explain.
  - (d) If he played in 81 games, in about how many games do you think he scored between 20 and 30 points?
40. In the 2010 Winter Olympics in Vancouver, the mean distance of ski jumpers in the final round on the normal hill was 100.5 meters, with a standard deviation of 3.3 meters. On the long hill, the mean distance of ski jumpers in the final round was 125 meters, with a standard deviation of 8.2 meters. Both distributions can be modeled by a Normal distribution.
- (a) Simon Ammann of Switzerland won both events, with jumps of 108 meters on the normal hill and 138 meters on the long hill. Which *PERFORMANCE* was better, relatively speaking? Justify your answer.
  - (b) On the normal hill, the middle 95% of the jumpers flew between what two distances?
  - (c) Estimate the proportion of jumpers that flew over 100 meters on the normal hill.
  - (d) To be at the 80th percentile of distances on the normal hill, about how far would a jumper need to fly?
41. Suppose that a football kicker can kick the ball an average of 61 yards with a standard deviation of 8 yards and that the distribution of distances can be modeled by a Normal distribution.
- (a) If the kickoff is from the 40-yard-line, the kick would need to travel 60 yards to reach the end zone. About what proportion of the time will a kick reach the end zone?
  - (b) If the kickoff is moved back 5 yards to the 35-yard-line, about what proportion of the time will a kick reach the end zone?

- (c) Suppose that a different kicker averages 52 yards and 10% of his kickoffs reach the end zone when he kicks off from the 40-yard-line. What is his standard deviation, assuming the distribution of his distances can be modeled by a Normal distribution?

### OTHER APPLICATIONS

42. According to the Centers for Disease Control and Prevention, the heights of three-year-old females can be modeled by a Normal distribution with a mean of 94.5 centimeters and a standard deviation of 4 centimeters.<sup>7</sup>
- (a) Sketch what this distribution should look like by drawing a Normal curve and labeling the mean, mean  $\pm 1$  SD, and mean  $\pm 2$  SD.
- (b) Calculate and interpret the z-score for Macy, who was 100 centimeters tall at age 3.
- (c) In a school that has 30 three-year-old females, about how many will be between 90 and 100 centimeters tall?
- (d) Macy's older brother, Brody, is 12 years old and 158 centimeters tall. The mean height for 12-year-old males is 149 centimeters, with a standard deviation of 7.9 centimeters. Who is taller, relatively speaking, Macy or Brody?
43. The May 2010 edition of *Consumer Reports* magazine reported the usable capacity of 36 side-by-side refrigerators. The mean capacity was 15.825 cubic feet, with a standard deviation of 1.217 cubic feet.
- (a) If the distribution of usable capacity can be modeled by a Normal distribution, about how many refrigerators will be within 1 standard deviation of the mean? About how many will be within 2 standard deviations of the mean?
- (b) The 36 measurements of usable capacity are shown below. Calculate the number of refrigerators with a usable capacity within 1 standard deviation of the mean and the number within 2 standard deviations of the mean:
- 12.9 13.7 14.1 14.2 14.5 14.5 14.6 14.7 15.1 15.2 15.3 15.3  
15.3 15.3 15.5 15.6 15.6 15.8 16.0 16.0 16.2 16.2 16.3 16.4  
16.5 16.6 16.6 16.6 16.8 17.0 17.0 17.2 17.4 17.4 17.9 18.4
- (c) Using your answers to parts (a) and (b), what can you say about the shape of the distribution of usable capacity?
44. In humans, the average length of a pregnancy is 266 days.<sup>8</sup> Suppose that the distribution of pregnancy lengths can be modeled by a Normal distribution with a standard deviation of 10 days.
- (a) About what proportion of pregnancies will last fewer than 270 days?
- (b) About what proportion of pregnancies will last longer than 250 days?
- (c) About what proportion of women will deliver their babies within 1 week of the due date?
- (d) What is the 30th percentile of the distribution of pregnancy lengths?

45. In a certain town, 20% of the residents are senior citizens. When a polling organization randomly selects samples of 500 residents for a telephone poll, the percentage of senior citizens in the poll can be modeled by a Normal distribution with a mean of 20% and a standard deviation of 1.8%. In one poll, 25% of the respondents were senior citizens.
- (a) Assuming the sample is really randomly selected, would it be surprising to get a percentage of senior citizens this high? Justify your response.
- (b) What are some possible explanations for why the percentage of senior citizens is higher than we expected?

## FOR INVESTIGATION

1. Do a historical comparison of at least three prominent sports *PERFORMANCES* from different eras. Make sure to pick a reasonable set of athletes to calculate the mean and standard deviation in each era.
2. Using athletes from your school, identify the "athlete of the year" by calculating and comparing z-scores for athletes in different sports.
3. Organize a scholar-athlete competition at your school as described in Exercise 12. Use z-scores to determine the winner.
4. Using a sport of your choice, create a fantasy league using at least three variables. Then use data from the most recent season to create a ranked list of the most valuable fantasy athletes.