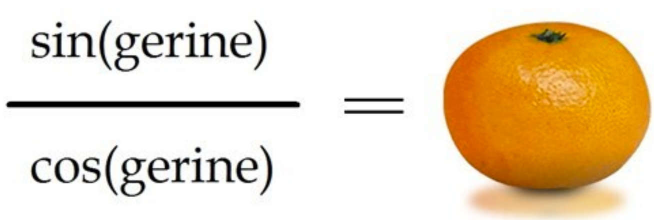
Name: ______________________________

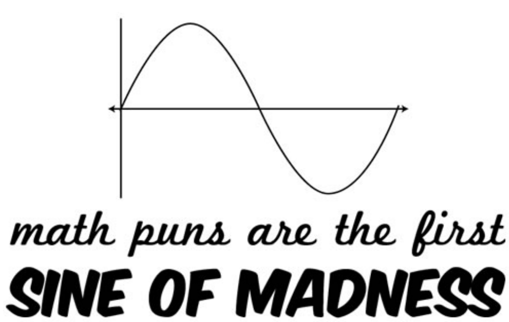
v. 6.0

Trig Packet

Trigonometry and Trigonometric Functions

I. AMC (Merrill) Practice Worksheets
II. FMC (Connally) Exercises and Problems
III. PTCA (Foerster) Explorations
IV. ASM2 (Brown) Study Guide, Exercises
V. N: Notes, formulas, other worksheets and exercises





NAME ______________________ DATE ____________

5-1

Practice Worksheet

Angles and Their Measure

If each angle has the given measure and is in standard position, determine the quadrant in which its terminal side lies.

1. $\frac{7\pi}{12}$ **2.** $-\frac{2\pi}{3}$ **3.** $371°$ **4.** $\frac{14\pi}{5}$

5. $-156°$ **6.** $1000°$ **7.** $332°$ **8.** $-240°$

Change each degree measure to radian measure in terms of π.

9. $36°$ **10.** $-250°$ **11.** $-145°$ **12.** $6°$

13. $870°$ **14.** $18°$ **15.** $-820°$ **16.** $345°$

Change each radian measure to degree measure.

17. -1 **18.** 4π **19.** -2.56 **20.** 12.85

21. $\frac{3\pi}{16}$ **22.** $-\frac{7\pi}{9}$ **23.** $\frac{13\pi}{30}$ **24.** $-\frac{17\pi}{3}$

Find one positive angle and one negative angle that are coterminal with each angle.

25. $70°$ **26.** $-\frac{2\pi}{5}$ **27.** $-300°$ **28.** $\frac{3\pi}{4}$

Find the reference angle for each angle with the given measure.

29. $-20°$ **30.** $160°$ **31.** $-545°$ **32.** $300°$

33. $\frac{10\pi}{3}$ **34.** $-\frac{5\pi}{8}$ **35.** $-\frac{\pi}{4}$ **36.** $-\frac{7\pi}{3}$

NAME ______________________ DATE ____________

5-2

Practice Worksheet

Central Angles and Arcs

Given the radian measure of a central angle, find the measure of its intercepted arc in terms of π in a circle of radius 10 cm.

1. $\frac{\pi}{6}$ **2.** $\frac{\pi}{3}$ **3.** $\frac{\pi}{2}$ **4.** $\frac{\pi}{5}$

5. $\frac{3\pi}{5}$ **6.** $\frac{4\pi}{7}$ **7.** $\frac{\pi}{12}$ **8.** $\frac{\pi}{24}$

Given the measurement of a central angle, find the measure of its intercepted arc in terms of π in a circle of diameter 60 in.

9. 10° **10.** 60° **11.** 42° **12.** 50°

13. 72° **14.** 110° **15.** 35° **16.** 65°

Given the measure of an arc, find the degree measure to the nearest tenth of the central angle it subtends in a circle of radius 16 cm.

17. 87 cm **18.** 5.6 cm **19.** 12 cm **20.** 25 cm

21. 10.24 cm **22.** 7.9 cm **23.** 11 cm **24.** 6 cm

Find the area of each sector to the nearest tenth, given its central angle, θ, and the radius of the circle.

25. $\theta = \frac{\pi}{6}$, $r = 14$ cm **26.** $\theta = \frac{\pi}{6}$, $r = 12$ ft

NAME ____________________ DATE __________

5-3

Practice Worksheet

Circular Functions

Find the values of the six trigonometric functions of an angle in standard position if the given point lies on its terminal side.

1. $(-1, 5)$ **2.** $(6, -8)$ **3.** $(3, 2)$ **4.** $(-3, -4)$

5. $(0, -4)$ **6.** $(7, 0)$ **7.** $(\sqrt{2}, -\sqrt{2})$ **8.** $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

Suppose θ is an angle in standard position whose terminal side lies in the given quadrant. For each function, find the values of the remaining five trigonometric functions of θ.

9. $\cos\theta = \frac{3}{5}$; quadrant I **10.** $\sin\theta = -\frac{2}{3}$; quadrant IV

NAME ________________________ DATE ____________

5-4

Practice Worksheet

Trigonometric Functions of Special Angles

Find each exact value. Do not use a calculator.

1. $\sin \frac{\pi}{4}$

2. $\cos \frac{\pi}{4}$

3. $\tan \frac{\pi}{4}$

4. $\cos 210°$

5. $\sin 300°$

6. $\tan 330°$

7. $\sin \frac{3\pi}{4}$

8. $\cos \frac{3\pi}{4}$

9. $\tan \frac{3\pi}{4}$

10. $\sin 90°$

11. $\csc 270°$

12. $\tan 45°$

13. $\cos \frac{3\pi}{2}$

14. $\tan \frac{3\pi}{2}$

15. $\sin \frac{3\pi}{2}$

Use a calculator to approximate each value to four decimal places.

16. $\cot (-75°)$

17. $\sin 634°$

18. $\cos 235°$

19. $\sin 2$

20. $\sec 4.28$

21. $\cot 0.23$

NAME ______________________ DATE ____________

6-2 Practice Worksheet

Amplitude, Period, and Phase Shift and Horizontal Shift.

State the amplitude, period, and phase shift and horizontal shift

1. $y = -2 \sin \theta$ **2.** $y = 10 \sec \theta$ **3.** $y = -3 \sin 4\theta$

4. $y = 0.5 \sin \left(\theta - \frac{\pi}{3}\right)$ **5.** $y = 2.5 \cos (\theta + 180°)$ **6.** $y = -1.5 \sin \left(4\theta - \frac{\pi}{4}\right)$

Write an equation of the sine function with each amplitude, period, and horizontal ***shift.***

7. amplitude = 0.75, period = 360°, horizontal shift = 30°

8. amplitude = 4, period = 3°, horizontal shift = –30°

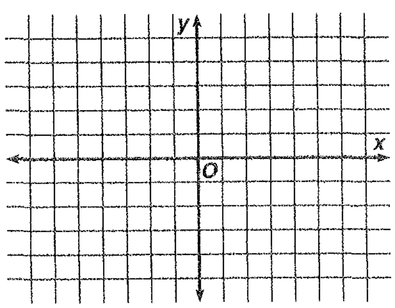
Write an equation of the cosine function with each amplitude, period, and phase shift.

9. amplitude = 3.75, period = 90°, phase shift = 4°

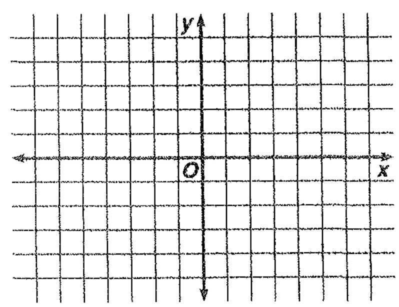
10. amplitude = 12, period = 45°, phase shift = 180°

Graph each function.

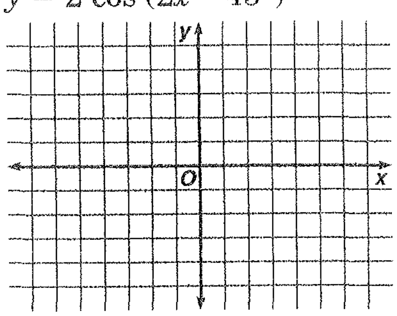
11. $y = 0.5 \sin x$



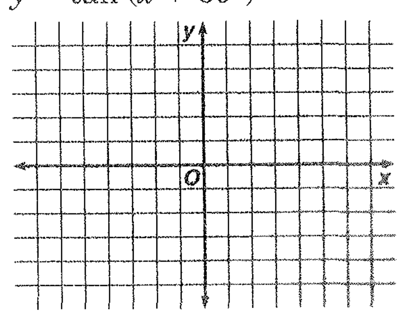
12. $y = 2 \cos (3x)$



13. $y = 2 \cos (2x - 45°)$



14. $y = \tan (x + 60°)$



6-4

NAME ____________________ DATE ____________

Practice Worksheet

Inverse Trigonometric Functions

Write each equation in the form of an inverse relation.

1. $0.75 = \sin x$

2. $-1 = \cos x$

3. $0.1 = \tan \theta$

4. $\frac{3}{5} = \cos x$

5. $\sin x = \frac{\sqrt{3}}{2}$

6. $\cos \alpha = \frac{12}{13}$

Find the values of x in the interval $0° \leq x \leq 360°$ that satisfy each equation.

7. $x = \arccos 1$

8. $\arccos \frac{\sqrt{2}}{2} = x$

9. $\arcsin \frac{1}{2} = x$

10. $\sin^{-1}(-1) = x$

11. $\sin^{-1} \frac{\sqrt{2}}{2} = x$

12. $\cot^{-1} 1 = x$

Evaluate each expression. Assume that all angles are in Quadrant I.

13. $\cos\left(\cos^{-1} \frac{1}{2}\right)$

14. $\sin\left(\cos^{-1} \frac{1}{2}\right)$

15. $\cos\left(\sin^{-1} \frac{1}{2}\right)$

16. $\tan\left(\sin^{-1} \frac{\sqrt{2}}{2} - \cos^{-1} \frac{\sqrt{2}}{2}\right)$

17. Verify that $\sin^{-1} \frac{\sqrt{3}}{2} + \sin^{-1} \frac{1}{2} = 90°$. Assume that all angles are in Quadrant I.

NAME ______________________ DATE ____________

7-1

Practice Worksheet

Basic Trigonometric Identities

Solve for values of θ between 0° and 90°.

Use Pythagorean Identities (not triangles)

1. If $\tan\theta = 2$, find $\cot\theta$.

2. If $\sin\theta = \frac{2}{3}$, find $\cos\theta$.

3. If $\cos\theta = \frac{1}{4}$, find $\tan\theta$

4. If $\tan\theta = 3$, find $\sec\theta$.

5. If $\sin\theta = \frac{7}{10}$, find $\cot\theta$.

6. If $\tan\theta = \frac{7}{2}$, find $\sin\theta$.

Express each value as a function of an angle in Quadrant I.

7. $\sin 458°$

8. $\cos 892°$

9. $\tan(-876°)$

10. $\csc 495°$

Simplify.

11. $\dfrac{\cot A}{\tan A}$

12. $\dfrac{\sin^2\beta\cot\beta}{\cos\beta}$

13. $\sin^2\theta\cos^2\theta - \cos^2\theta$

14. $\cos x + \sin x\tan x$

15. $\dfrac{1}{\sin^2\theta} - \dfrac{\cos^2\theta}{\sin^2\theta}$

16. $\dfrac{\cos^2\theta}{1+\sin\theta}$

*17. $\dfrac{\tan^2\theta - 1}{\tan^2\theta + 1} + 2\cos^2\theta$

*18. $1 - \dfrac{\sin^2\theta}{1+\cos\theta}$

7-2

NAME ______________________ DATE ______________

Practice Worksheet

Verifying Trigonometric Identities

Verify that each of the following is an identity.

1. $\frac{\csc x}{\cot x + \tan x} = \cos x$

2. $\sin^3 x - \cos^3 x = (1 + \sin x \cos x)(\sin x - \cos x)$

3. $\frac{1}{\sin y - 1} - \frac{1}{\sin y + 1} = -2\sec^2 y$

4. $1 - 2\sin^2 r + \sin^4 r = \cos^4 r$

5. $\tan u + \frac{\cos u}{1 + \sin u} = \sec u$

6. $\frac{\tan x + \sec x}{\sec x - \cos x + \tan x} = \csc x$

Find a numerical value of one trigonometric function of each x.

7. $\sin x = 3\cos x$

8. $\cos x = \cot x$

Exercises and Problems for Section 6.1

FMC 6.1 A

Exercises

In Exercises 1–8, do the functions appear to be periodic with period less than 4?

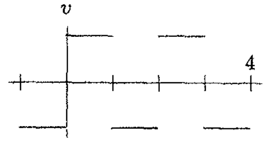
1.

t	0	1	2	3	4	5	6
$f(t)$	1	5	7	1	5	7	1

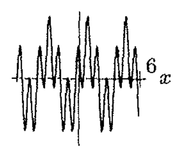
2.

r	0	π	2π	3π	4π	5π	6π	7π
$q(r)$	0	1	0	-1	1	0	1	0

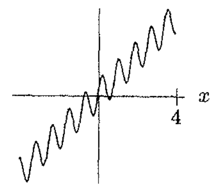
3.



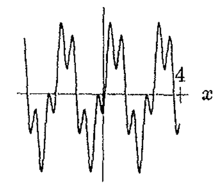
4.



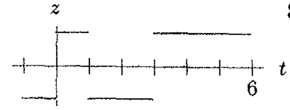
5.



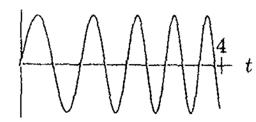
6.



7.



8.



In Exercises 9–12, estimate the period of the periodic functions.

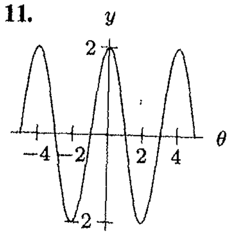
9.

t	0	1	2	3	4	5	6
$f(t)$	12	13	14	12	13	14	12

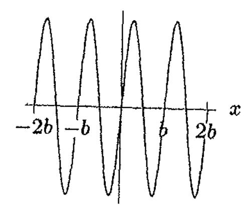
10.

z	1	11	21	31	41	51	61	71	81
$g(z)$	5	3	2	3	5	3	2	3	5

11.



12.



30. Table 6.3 gives the height $h = f(t)$ in feet of a weight on a spring where t is time in seconds. Find the midline, amplitude and period of the function f.

Table 6.3 MAKE A GRAPH

t	0	1	2	3	4	5	6	7
h	4.0	5.2	6.2	6.5	6.2	5.2	4.0	2.8
t	8	9	10	11	12	13	14	15
h	1.8	1.5	1.8	2.8	4.0	5.2	6.2	6.5

Problems 23–26 concern a weight suspended from the ceiling by a spring. (See Figure 6.8.) Let d be the distance in centimeters from the ceiling to the weight. When the weight is motionless, $d = 10$. If the weight is disturbed, it begins to bob up and down, or *oscillate*. Then d is a periodic function of t, time in seconds, so $d = f(t)$.

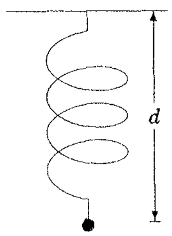


Figure 6.8

23. Determine the midline, period, amplitude, and the minimum and maximum values of f from the graph in Figure 6.9. Interpret these quantities physically; that is, use them to describe the motion of the weight.

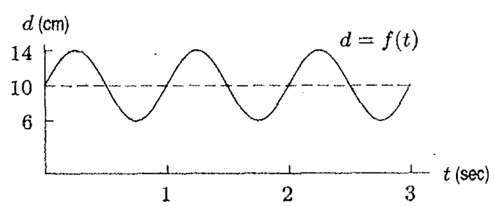


Figure 6.9

24. A new experiment with the same weight and spring is represented by Figure 6.10. Compare Figure 6.10 to Figure 6.9. How do the oscillations differ? For both figures, the weight was disturbed at time $t = -0.25$ and then left to move naturally; determine the nature of the initial disturbances.

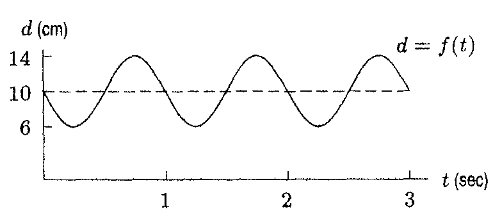


Figure 6.10

25. The weight in Problem 23 is gently pulled down to a distance of 14 cm from the ceiling and released at time $t = 0$. Sketch its motion for $0 \leq t \leq 3$.

26. Figures 6.11 and 6.12 describe the motion of two different weights, A and B, attached to two different springs. Based on these graphs, which weight:

(a) Is closest to the ceiling when not in motion?
(b) Makes the largest oscillations?
(c) Makes the fastest oscillations?

NOTE VERTICAL SCALES

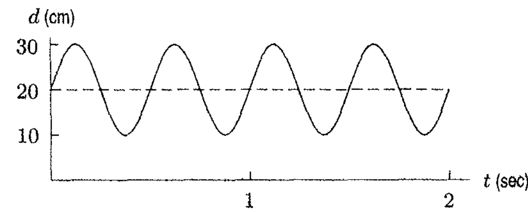


Figure 6.11: Weight A

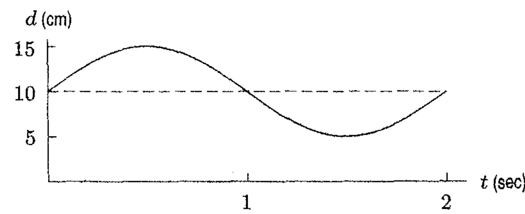


Figure 6.12: Weight B

FMC 6.2A

28. Find an angle ϕ, with $0° < \phi < 360°$, that has the same
(a) Cosine as $53°$ (b) Sine as $53°$

30. A revolving door (which rotates counterclockwise in Figure 6.28) was designed with five equally spaced panels for the entrance to the Pentagon. The arcs BC and AD have equal length.

(a) What is the angle between two adjacent panels?

(b) A four-star general enters by pushing on the panel at point B, and leaves the panel at point D. What is the angle of rotation?

(c) With the door in the position shown in Figure 6.28, an admiral leaves the Pentagon by pushing the panel between A and D to point B. What is the angle of rotation?

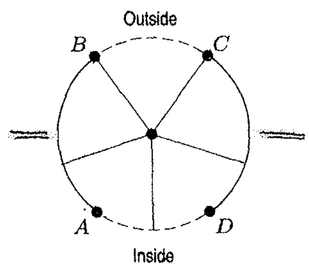


Figure 6.28

31. For the angle ϕ shown in Figure 6.29, sketch each of the following angles.

(a) $180+\phi$ **(b)** $180-\phi$ **(c)** $90-\phi$ **(d)** $360-\phi$

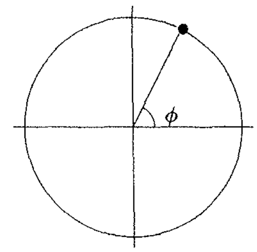


Figure 6.29

32. Let θ be an angle in the first quadrant, and suppose $\sin\theta = a$. Evaluate the following expressions in terms of a. (See Figure 6.30.)

(a) $\sin(\theta + 360°)$ **(b)** $\sin(\theta + 180°)$
(c) $\cos(90° - \theta)$ **(d)** $\sin(180° - \theta)$
(e) $\sin(360° - \theta)$ **(f)** $\cos(270° - \theta)$

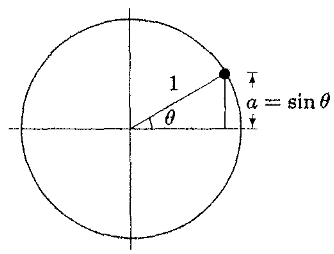


Figure 6.30

FMC 6.3A

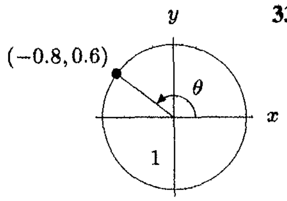
29. Without using a calculator, give the sign of each of the following numbers:

(a) $\cos 3$ **(b)** $\sin 4$ **(c)** $\sin(-4)$ **(d)** $\cos 7$

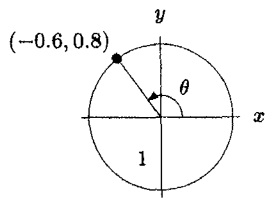
38. How far does the tip of the minute hand of a clock move in 35 minutes if the hand is 6 inches long?

Evaluate $\sin\theta$ and $\cos\theta$ for the angle θ on the unit circle in Problems 32–33.

32.



33.



34. An ant starts at the point $(1, 0)$ on the unit circle and walks counterclockwise a distance of 3 units around the circle. Find the x and y coordinates (accurate to 2 decimal places) of the final location of the ant.

44. Do you think there is a value of t for which $\cos t = t$? If
a) so, estimate the value of t. If not, explain why not.

USE RADIANS.

36. For ϕ in Figure 6.43, sketch the following angles.

(a) $\pi + \phi$ **(b)** $\pi - \phi$
(c) $\pi/2 - \phi$ **(d)** $2\pi - \phi$

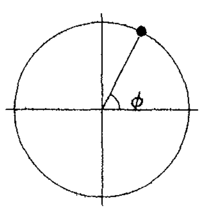
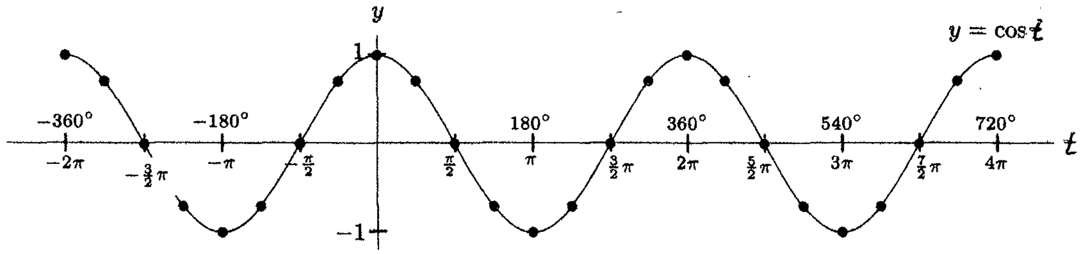


Figure 6.43

44b, What about $f(t) = \frac{\sin(t)}{t}$?
— does this have a value at $t=0$?
— What about "near" $t=0$?

FMC 6.4A



27. **(a)** Match the lengths p, q, r, s marked on the unit circle in Figure 6.55 with the following values:

(i) $t = 0.8$ (ii) $t = \pi - 2.9$
(iii) $\cos(0.8)$ (iv) $-\cos(2.9)$

(b) On a graph of $y = \cos t$, sketch segments corresponding to each of the values in part (a).

graph

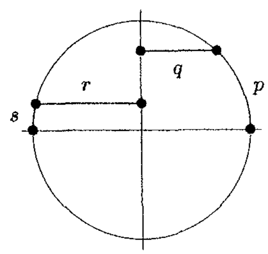


Figure 6.55

28. **(a)** Write an expression for the slope of the line segment joining P and Q in Figure 6.56.
(b) Evaluate your expression for $a = \pi/4$, $b = 4\pi/3$. Give an exact value for your answer.

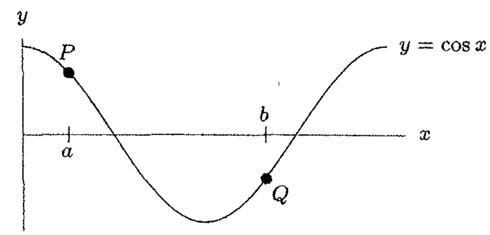


Figure 6.56

16. Match each of the letters A–G in Figure 6.49 to one of the following values of x (in radians): $1, 2, 4, 5, \pi/2, \pi,$ and $3\pi/2$.

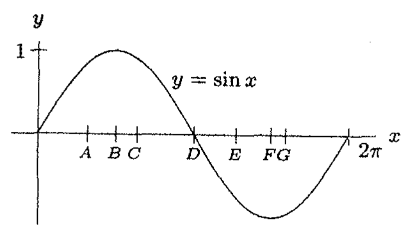


Figure 6.49

20. Figure 6.53 shows $y = \sin(x - \frac{\pi}{2})$ and $y = \sin(x + \frac{\pi}{2})$ starting at $x = 0$. Identify which is which.

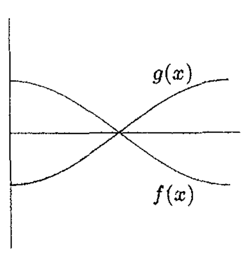


Figure 6.53

30. A circle of radius 5 is centered at the point $(-6, 7)$. Find a formula for $f(\theta)$, the x-coordinate of the point P in Figure 6.58.

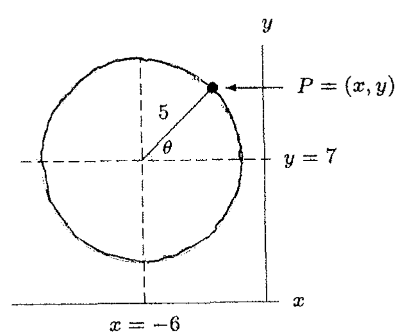


Figure 6.58

FMC 6.5A

In Exercises 1–4, state the period, amplitude, and midline.

1. $y = 7\sin(4(t+7)) - 8$

2. $y = 6\sin(t+4)$

3. $y = \pi\cos(2t+4) - 1$

4. $2y = \cos(8(t-6)) + 2$

Also state the horizontal shift and phase shift. (See below)

32. Find a possible formula for the trigonometric function whose values are in the following table.

x	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1
$g(x)$	2	2.6	3	3	2.6	2	1.4	1	1	1.4	2

38. The pressure, P (in lbs/ft^2), in a pipe varies over time. Five times an hour, the pressure oscillates from a low of 90 to a high of 230 and then back to a low 90. The pressure at $t = 0$ is 90.

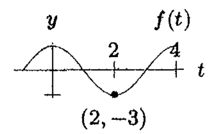
(a) Graph $P = f(t)$, where t is time in minutes. Label your axes.

(b) Find a possible formula for $P = f(t)$.

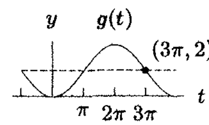
(c) By graphing $P = f(t)$ for $0 \le t \le 2$, estimate when the pressure first equals 115 lbs/ft^2.

In Exercises 13–20, find formulas for the trigonometric functions.

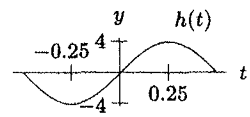
13.



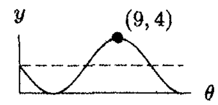
14.



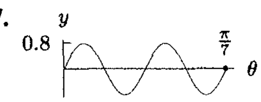
15.



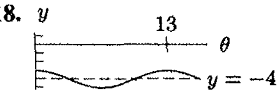
16.



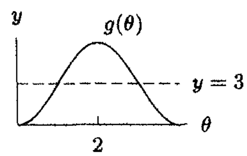
17.



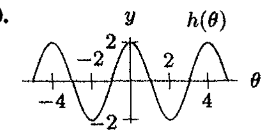
18.



19.



20.



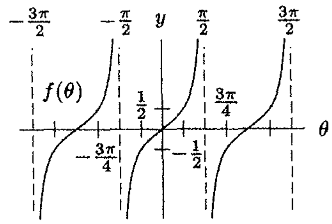
$$y = A\sin(B(t-h)) + k,$$
$$y = A\cos(B(t-h)) + k.$$

Amplitude $= |A|$; period $= 2\pi/|B|$; midline $y = k$.
Phase shift $= Bh$; horizontal shift $= h$.

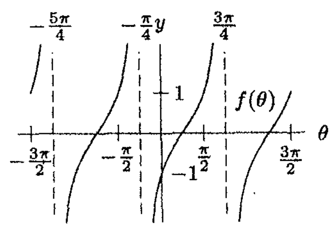
FMC 6.6A

In Exercises 16–17, give a possible formula for the function.

16.



17.



30. **(a)** Find an equation for the line l in Figure 6.76.
(b) Find the x-intercept of the line.

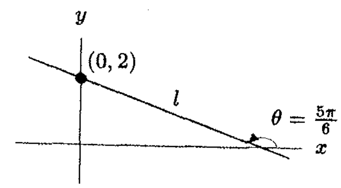
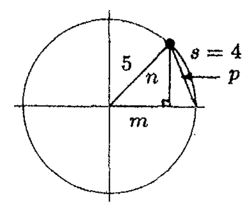


Figure 6.76

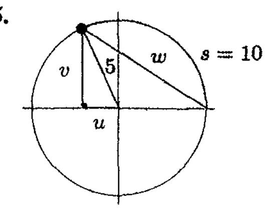
22. **(a)** $\cos\alpha = -\sqrt{3}/5$ and α is in the third quadrant. Find exact values for $\sin\alpha$ and $\tan\alpha$.
(b) $\tan\beta = 4/3$ and β is in the third quadrant. Find exact values for $\sin\beta$ and $\cos\beta$.

Find exact values for the lengths of the labeled segments in Problems 34–35.

34.



35.



Problems 26–29 give an expression for one of the three functions $\sin\theta$, $\cos\theta$, or $\tan\theta$, with θ in the first quadrant. Find expressions for the other two functions. Your answers will be algebraic expressions in terms of x.

26. $\sin\theta = x/3$

27. $\cos\theta = 4/x$

28. $x = 2\cos\theta$

29. $x = 9\tan\theta$

FMC 6.7A

36. In your own words, explain what each of the following expressions means. Evaluate each expression for $x = 0.5$. Give an exact answer if possible.

(a) $\sin^{-1} x$ **(b)** $\sin(x^{-1})$ **(c)** $(\sin x)^{-1}$.

Solve the equations in Problems 29–32 for $0 \le t \le 2\pi$. First estimate answers from a graph; then find exact answers.

29. $\cos(2t) = \frac{1}{2}$ **30.** $\tan t = \frac{1}{\tan t}$

31. $2\sin t \cos t - \cos t = 0$ **32.** $3\cos^2 t = \sin^2 t$

35. Approximate the x-coordinates of points P and Q shown in Figure 6.90, assuming that the curve is a sine curve. [Hint: Find a formula for the curve.]

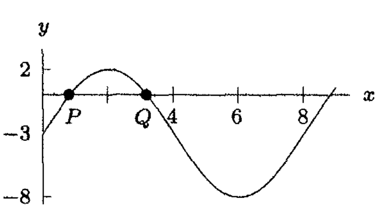


Figure 6.90

44. You are perched in the crow's nest, C, on top of the mast of a ship, S. See Figure 6.91. You will calculate how far you can see when you are x meters above the surface of the ocean.

(a) Find formulas for d, the distance you can see to the horizon, H, and l, the distance to the horizon along the earth's surface, in terms of x, the height of the ship's mast, and r, the radius of the earth.

(b) How far is the horizon from the top of a 50-meter mast? How far, measured along the earth's surface, is the horizon from the ship's position on the ocean? Use $r = 6{,}370{,}000$ meters.

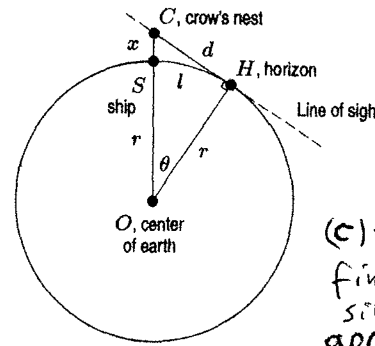


Figure 6.91

(c) Try to find a simple approximate formula

45. Let k be a positive constant and t be an angle measured in radians. Consider the equation

$$k \sin t = t^2.$$

(a) Explain why any solution to the equation must be between $-\sqrt{k}$ and $\sqrt{k}$, inclusive.

(b) Approximate every solution to the equation when $k = 2$.

(c) Explain why the equation has more solutions for larger values of k than it does for small values.

(d) Approximate the least value of k, if any, for which the equation has a negative solution.

FMC7.5A

Convert the polar coordinates in Exercises 14–17 to Cartesian coordinates. Give exact answers.

14. $(1, 2\pi/3)$ **15.** $(\sqrt{3}, -3\pi/4)$

16. $(2\sqrt{3}, -\pi/6)$ **17.** $(2, 5\pi/6)$

Convert the Cartesian coordinates in Problems 18–21 to polar coordinates.

18. $(1,1)$ **19.** $(-1,0)$

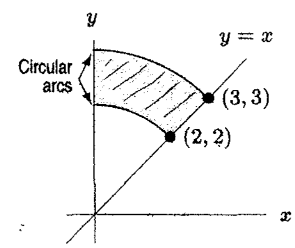
20. $(\sqrt{6}, -\sqrt{2})$ **21.** $(-\sqrt{3}, 1)$

For Problems 22–28, the origin is at the center of a clock, with the positive x-axis going through 3 and the positive y-axis going through 12. The hour hand is 3 cm long and the minute hand is 4 cm long. What are the Cartesian coordinates and polar coordinates of the tips of the hour hand and minute hand, H and M, respectively, at the following times?

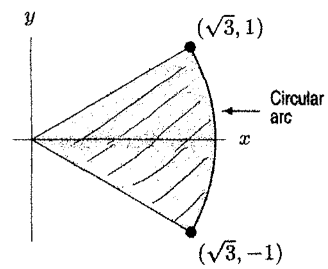
22. 12 noon **23.** 3 pm **24.** 9 am **25.** 1:30 pm
26. 7 am **27.** 3:30 pm **28.** 9:15 am

In Problems 29–31, give inequalities for r and θ which describe the following regions in polar coordinates.

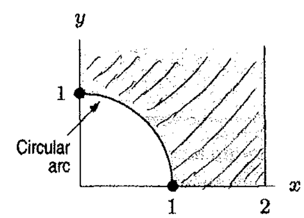
29.



30.



31.



Note: Region extends indefinitely in the y-direction.

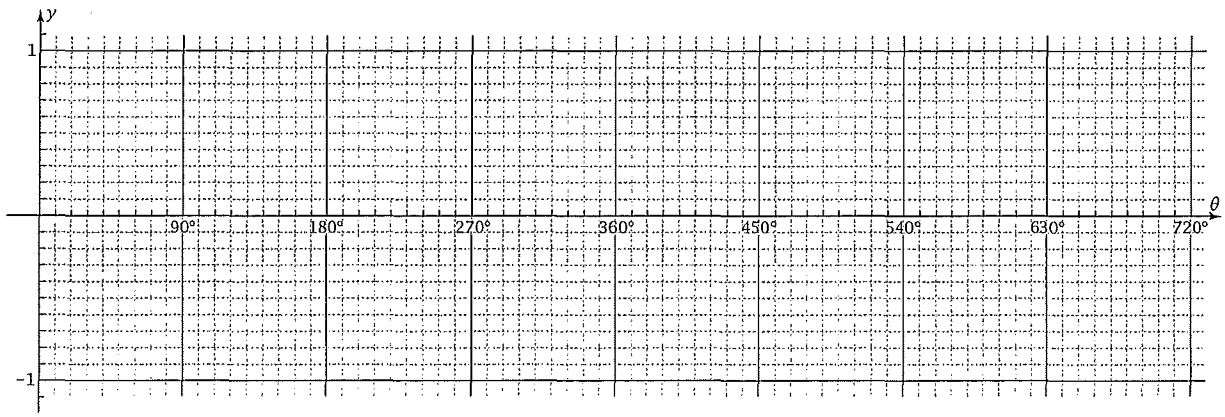
Name: ____________ Group Members: ____________

Exploration 3-1b: Sine and Cosine Graphs, Manually

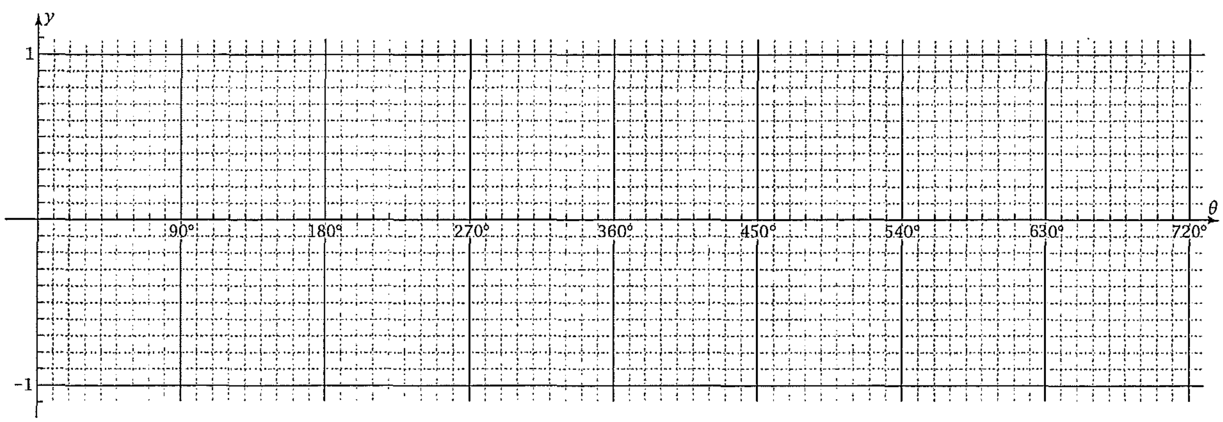
Date: ____________

Objective: Find the shape of sine and cosine graphs by plotting them on graph paper.

1. On your grapher, make a table of values of $y = \sin\theta$ for each 10° from 0° to 90°. Set the mode to round to 2 decimal places. Plot the values on this graph paper. Also plot $y = \sin\theta$ for each 90° through 720°. Connect the points with a smooth curve, observing the shape you plotted for 0° to 90°.



2. Plot the graph of $y = \cos\theta$ pointwise, the way you did for sine in Problem 1.



3. Find sin 45° and cos 65°. Show that the corresponding points are on the graphs in Problems 1 and 2, respectively.

4. Find the inverse trigonometric functions $\theta = \sin^{-1} 0.4$ and $\theta = \cos^{-1} 0.8$ Show that the corresponding points are on the graphs in Problems 1 and 2, respectively.

5. What are the ranges of the sine and cosine functions?

6. Name a real-world situation where variables are related by a periodic graph like sine or cosine.

7. What did you learn as a result of doing this Exploration that you did not know before?

Name: ____________ Group Members: ____________

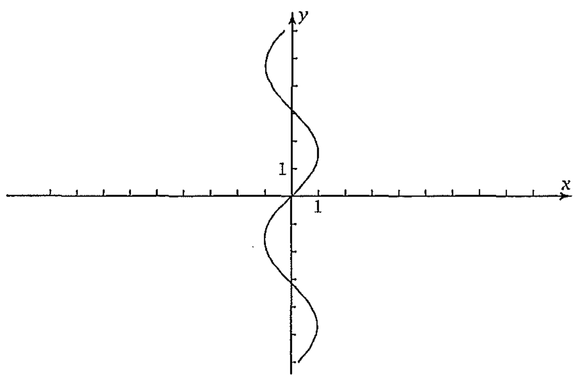
Exploration 4-6b: Principal Branches of Inverse Trigonometric Relations

Date: ________

Objective: Figure out the principal branches of each of the six inverse circular functions.

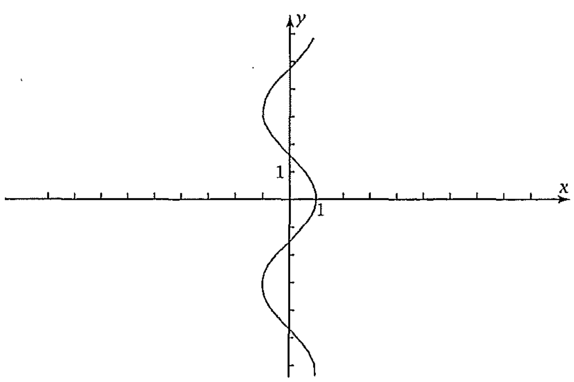
1. On your grapher, plot the inverse circular function $y = \sin^{-1} x$. Use equal scales on the two axes. On the graph of $y = \arcsin x$ shown here, darken the **principal branch** of the inverse sine relation on the part of the graph that is the inverse sine function. Give the range of the principal branch.

   Range: ____________



2. Use the technique in Problem 1 to find the range of $y = \cos^{-1} x$. Darken the principal branch on the graph of $y = \arccos x$, shown next.

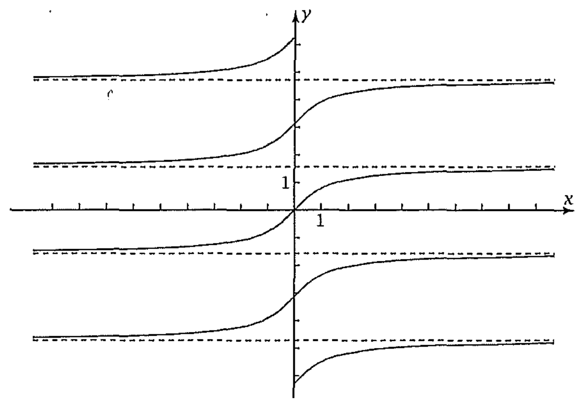
   Range: ____________



3. Why can't the range of the inverse cosine function ($y = \cos^{-1} x$) be the same as the range of the inverse sine function ($y = \sin^{-1} x$)?

4. Use the technique in Problems 1 and 2 to find the range of $y = \tan^{-1} x$. Darken the principal branch on this graph of $y = \arctan x$.

   Range: ____________



5. The range of $y = \sin^{-1} x$ is the closed interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Explain why the range of $y = \tan^{-1} x$ cannot include the endpoints.

6. If the range of the inverse tangent function ($y = \tan^{-1} x$) were $[0, \pi]$ (excluding $\frac{\pi}{2}$), like the range of $y = \cos^{-1} x$, then $y = \tan^{-1} x$ would still be a function. What disadvantage would there be to defining the range of $y = \tan^{-1} x$ this way?

(Over)

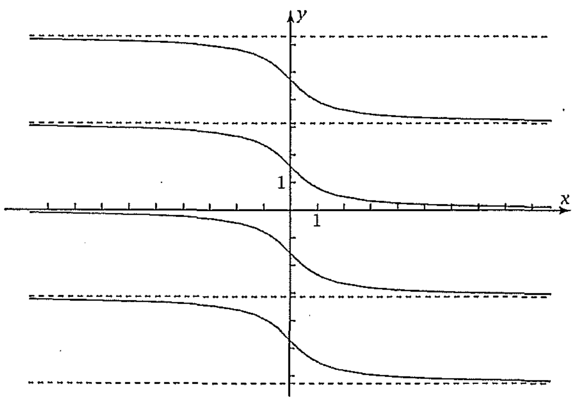
Exploration 4-6b: Principal Branches of Inverse Trigonometric Relations *continued*

Date: ____________

7. Duplicate the previous graph of $y = \arctan x$ on your grapher. Give the parametric equations you used. Check your graph with your instructor. ____________

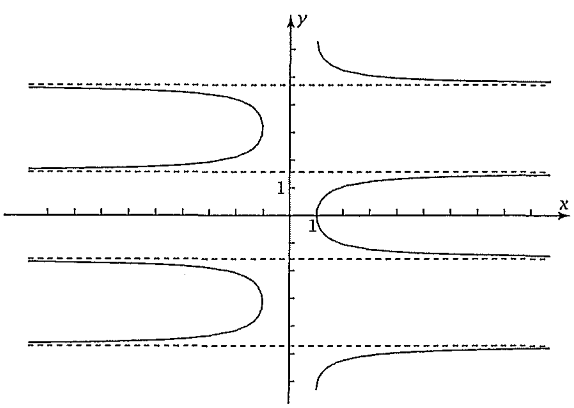
8. The graph here shows $y = \text{arccot}\, x$. How can you define the range of the function $y = \cot^{-1} x$ in such a way that the function is **continuous**? Darken this principal branch of $y = \text{arccot}\, x$.

   Range: ____________



9. This next graph shows $y = \text{arcsec}\, x$. There is no way to restrict the range to make a continuous function $y = \sec^{-1} x$ and still use all of the domain. Darken what you think would be the best choice for the principal branch. Write the range.

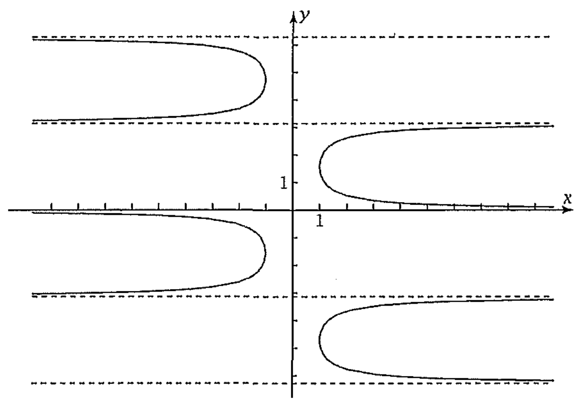
   Range: ____________



10. Look in the text to find out if the principal branch you chose in Problem 9 is the commonly accepted one. ____________

11. This next graph shows $y = \text{arccsc}\, x$. Shade what you think the principal branch is. Write the range of the function you shaded.

   Range: ____________



12. Does your answer to Problem 11 agree with the range listed in the text?

13. Look up in the text the five criteria for picking the ranges of the principal branches of the six inverse circular functions. Write the criteria here.

14. What did you learn as a result of doing this Exploration that you did not know before?

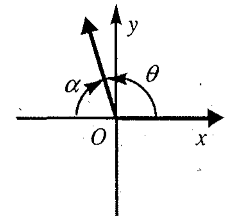
NAME ______________________________ DATE ______________

12–3 Trigonometric Functions of General Angles

Objective: To define trigonometric functions of general angles.

Vocabulary

Reference angle If θ is not a quadrantal angle, there is a unique acute angle α, corresponding to θ, such that either $\theta + \alpha$ or $\theta - \alpha$ is an integral multiple of 180°. α is called the reference angle of θ. Example: If θ measures 115°, its reference angle, α, measures 65°.



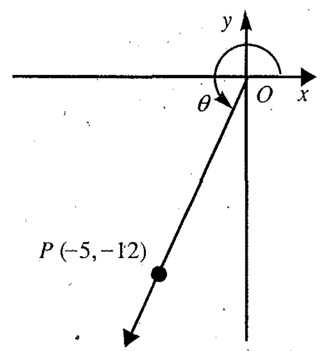
Formulas For any angle θ and any point $P(x, y)$ on the terminal side of θ:

$\sin\theta = \dfrac{y}{r}$ $\qquad$ $\cos\theta = \dfrac{x}{r}$ $\qquad$ $\tan\theta = \dfrac{y}{x}$, if $x \neq 0$

$\csc\theta = \dfrac{r}{y}$, if $y \neq 0$ $\qquad$ $\sec\theta = \dfrac{r}{x}$, if $x \neq 0$ $\qquad$ $\cot\theta = \dfrac{x}{y}$, if $y \neq 0$

Example 1 Find the values of the six trigonometric functions of an angle θ in standard position whose terminal side passes through $(-5, -12)$.

Solution First make a sketch.
Here $x = -5$ and $y = -12$.
Since r is the distance between (0, 0) and $(-5, -12)$, you can find r using the distance formula.



$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$r = \sqrt{(-5 - 0)^2 + (-12 - 0)^2}$$

$$r = \sqrt{25 + 144} = \sqrt{169} = 13$$

$\sin\theta = -\dfrac{12}{13}$ $\qquad$ $\cos\theta = -\dfrac{5}{13}$ $\qquad$ $\tan\theta = \dfrac{-12}{-5} = \dfrac{12}{5}$

$\csc\theta = -\dfrac{13}{12}$ $\qquad$ $\sec\theta = -\dfrac{13}{5}$ $\qquad$ $\cot\theta = \dfrac{-5}{-12} = \dfrac{5}{12}$

Find the values of the six trigonometric functions of an angle θ in standard position whose terminal side passes through point P. If any value is undefined, say so.

1. $P(4, -3)$ $\qquad$ 2. $P(-12, 5)$ $\qquad$ 3. $P(-2, -2)$ $\qquad$ 4. $P(15, -8)$ $\qquad$ 5. $P(0, 1)$

Example 2 Find the measure of the reference angle α for each given angle θ.

a. $\theta = 155°$ $\qquad$ **b.** $\theta = 310°30'$ $\qquad$ **c.** $\theta = -125°$

Solution

a. Since θ is in quadrant II: $\alpha = 180° - \theta = 180° - 155° = 25°$

b. Since θ is in quadrant IV:
$\alpha = 360° - \theta = 360° - 310°30' = 359°60' - 310°30' = 49°30'$

c. First find the positive angle that is coterminal with $-125°$:
$$-125° + 360° = 235°$$
Since 235° is in quadrant III: $\alpha = \theta - 180° = 235° - 180° = 55°$

NAME ______________________________ DATE ______________

12–3 Trigonometric Functions of General Angles *(continued)*

Find the measure of the reference angle α of the given angle θ.

6. $\theta = 250°$	**7.** $\theta = 172°$	**8.** $\theta = -200°$	**9.** $\theta = -142°$
10. $\theta = 730°$	**11.** $\theta = 480°$	**12.** $\theta = 98.2°$	**13.** $\theta = -166.2°$
14. $\theta = -288.6°$	**15.** $\theta = 149°20'$	**16.** $\theta = 243°15'$	**17.** $\theta = 300°50'$

Example 3 Write sin 230° as a function of an acute angle.

Solution The reference angle of 230° is $230° - 180° = 50°$.
Since 230° is in the third quadrant, its sine is negative (see note below).
$\therefore \sin 230° = -\sin 50°$.

Note: The signs of the trigonometric functions vary with the quadrants. All values are positive in the first quadrant. Sine and cosecant are positive in the second quadrant. Tangent and cotangent are positive in the third quadrant. Cosine and secant are positive in the fourth quadrant.

Write each of the following as a function of an acute angle.

18. $\sin 312°$	**19.** $\cos 215°$	**20.** $\tan(-163°)$	**21.** $\sin(-29°)$
22. $\sec 276.8°$	**23.** $\csc(-198.3°)$	**24.** $\cot 312°15'$	**25.** $\cos 259°12'$

Example 4 Find the exact values of the six trigonometric functions of 225°. Use the table of exact trigonometric values found in Lesson 12-2 of your textbook.

Solution The reference angle of 225° is 45°.

Since 225° is in the third quadrant, only the tangent and cotangent are positive.

$\sin 225° = -\sin 45° = -\frac{\sqrt{2}}{2}$ $\qquad$ $\csc 225° = -\csc 45° = -\sqrt{2}$

$\cos 225° = -\cos 45° = -\frac{\sqrt{2}}{2}$ $\qquad$ $\sec 225° = -\sec 45° = -\sqrt{2}$

$\tan 225° = \tan 45° = 1$ $\qquad$ $\cot 225° = \cot 45° = 1$

Find the exact values of the six trigonometric functions of each angle.

26. 120° **27.** −135° **28.** 210° **29.** −225° **30.** 420°

Mixed Review Exercises

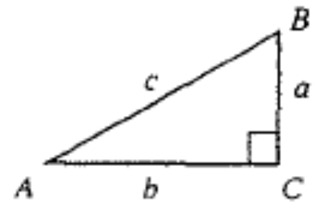
Find the zeros of each function. If the function has no zeros, say so.

1. $f(x) = 2x + 6$	**2.** $g(x) = \log(x - 2)$	**3.** $h(x) = 6x^2 + x - 2$
4. $F(x) = \dfrac{x^3 - 9x}{x^2 + 2}$	**5.** $G(x) = x^2 + 3x + 8$	**6.** $H(x) = 2^x$

Sheet 623: Right Angle Trig and Word Problems ASM2 p. 577–579

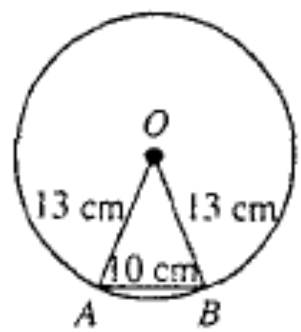
Find all missing sides (to two decimals) and angles (to one decimal).

Solve each right triangle ABC.

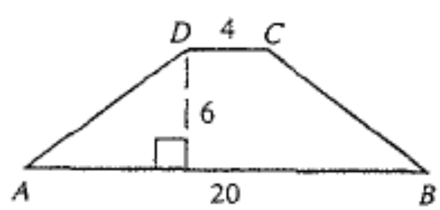


1. $\angle A = 36.2°$, $c = 68$
2. $\angle B = 15.8°$, $c = 12.2$
3. $\angle B = 65.4°$, $a = 2.35$
4. $\angle A = 82.1°$, $b = 246$
5. $\angle B = 48.3°$, $b = 74.7$
6. $\angle A = 24.0°$, $a = 5.25$
7. $a = 230$, $c = 320$
8. $a = 52.5$, $b = 28.0$
9. $a = 0.123$, $b = 0.315$
10. $b = 3.90$, $c = 42.5$

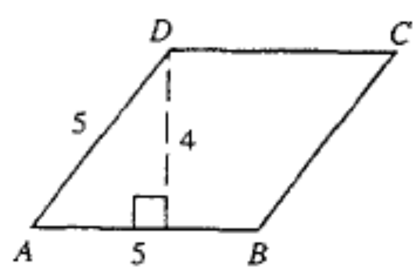
17. The radius of circle O is 13 cm and the length of $\overline{AB}$ is 10 cm. Find the measure of $\angle AOB$.



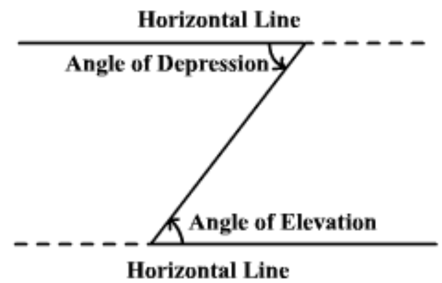
18. The height of an isosceles trapezoid is 6 units and the bases have lengths 4 units and 20 units. Find the measures of the angles.



19. A rhombus has sides 5 units long and its height is 4 units. Find its angles.



1. What is the angle of elevation of the sun when a tree 6.25 m tall casts a shadow 10.1 m long?

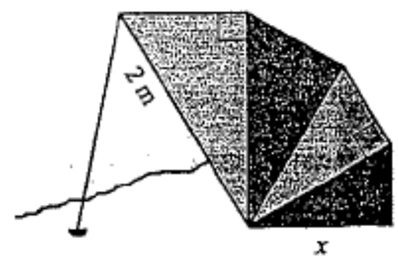


5. The angle of elevation of the summit of a mountain from the bottom of a ski lift is 33°. A skier rides 1000 ft on this ski lift to get to the summit. Find the vertical distance between the bottom of the ski lift and the summit.

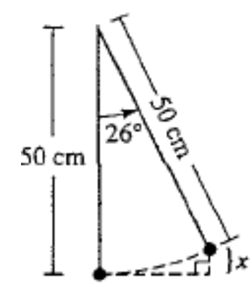
9. A pendulum in a grandfather clock is 160 cm long. The horizontal distance between the farthest points in a complete swing is 65 cm. Through what angle does the pendulum swing?

11. From the top of a 135 ft observation tower, a park ranger sights two forest fires on opposite sides of the tower. If their angles of depression are 42.5° and 32.6°, how far apart are the fires?

13. The side of the beach shelter shown at the right is made up of four 30°–60°–90° triangles. Find the dimension marked x.



14. A pendulum 50 cm long is moved 26° from the vertical. How much is the lower end of the pendulum raised?



17. In a molecule of carbon tetrachloride, the four chlorine atoms are at the vertices of a regular tetrahedron, with the carbon atom in the center. What is the angle between two of the carbon-chlorine bonds (shown in **bold lines** in the figure)?

Sheet # 622: Right Triangles

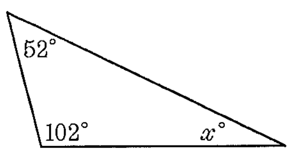
Name ____________________ Per/Sec. ____________

1. Find the distance between $(-5,6)$ and $(-9,6)$.

2. Find the distance between $(-3,2)$ and $(-3,-5)$.

3. Find the distance between $(3,-5)$ and $(-1,2)$.

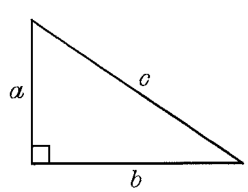
4. Find the value of x in the diagram.



5. In a right triangle, the side opposite the right angle is called the _____.

6. In a right triangle, the sides that form the right angle are called the _____.

7. In the diagram, $a = 3$ and $b = 4$. Find c.

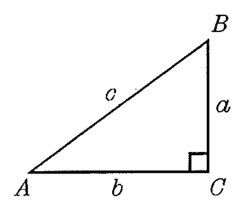


8. In the diagram, $a = 7$ and $b = 24$. Find c.

9. If a 25-foot ladder is placed against a wall so that it reaches a height of 24 feet, how far away from the base of the wall are the feet of the ladder?

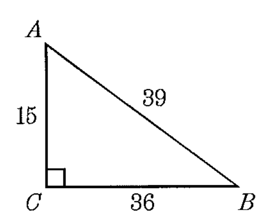
10. Two vehicles leave the same town at 8 am. One travels north at 30 mph, the other travels west at 45 mph. To the nearest hundredth of a mile, how far apart are they at 11 am the same day?

11. What is the sine ratio of $\angle A$?

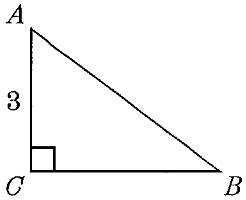


12. What is the cosine ratio of $\angle A$?

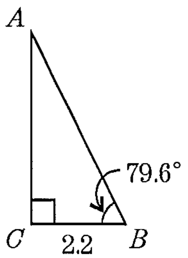
13. Given $\triangle ABC$ shown, express the *sine*, *cosine*, and *tangent* of $\angle A$ as reduced fractions.



14. If $\cos \angle A = \frac{3}{5}$, find AB.



15. Find AB to the nearest tenth.



16. Use a calculator to find the values of the following ratios to four decimal places.

a) $\sin 10°$

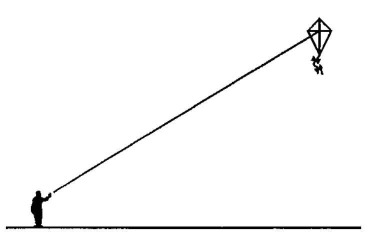
b) $\cos 80°$

c) $\tan 50°$

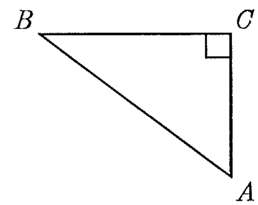
d) $\sin 65°$

e) $\tan 40°$

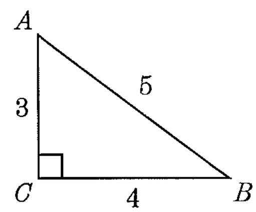
17. When I went kite flying the other day, I managed to let out an entire roll of string (400 feet). If the string, when pulled tight, formed a 40° angle with the ground, about how high was the kite?



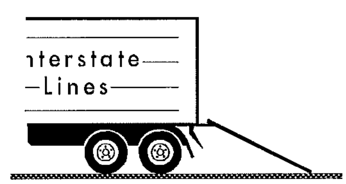
18. Solve the right triangle if $\angle A = 63°$ and $a = 9.7$ meters. Give lengths to 3 significant figures and angles to the nearest tenth of a degree.



19. $\sin \angle$_____ $= \frac{4}{5}$



20. One end of a ramp is raised to the back of a truck 1 meter above the ground. If the length of the ramp is 3 meters, what is the approximate measure of the angle the ramp makes with the ground? Round your answer to the nearest tenth of a degree.



Answer List

1. 4
2. 7
3. $\sqrt{65}$
4. 26
5. hypotenuse
6. legs
7. 5
8. 25
9. 7 ft
10. 162.25 miles
11. $\frac{a}{c}$
12. $\frac{b}{c}$
13. $\sin A = \frac{12}{13}$; $\cos A = \frac{5}{13}$; $\tan A = \frac{12}{5}$
14. 5
15. 12.2
16. 0.1736; 0.1736; 1.1918; 0.9063; 0.8391
17. ≈ 257.1 ft
18. $\angle B = 27.0°$, $b \approx 4.94$ m, $c \approx 10.9$ m
19. A
20. 19.5°

Name:____________________

Sketching Angles, Circles and Arcs

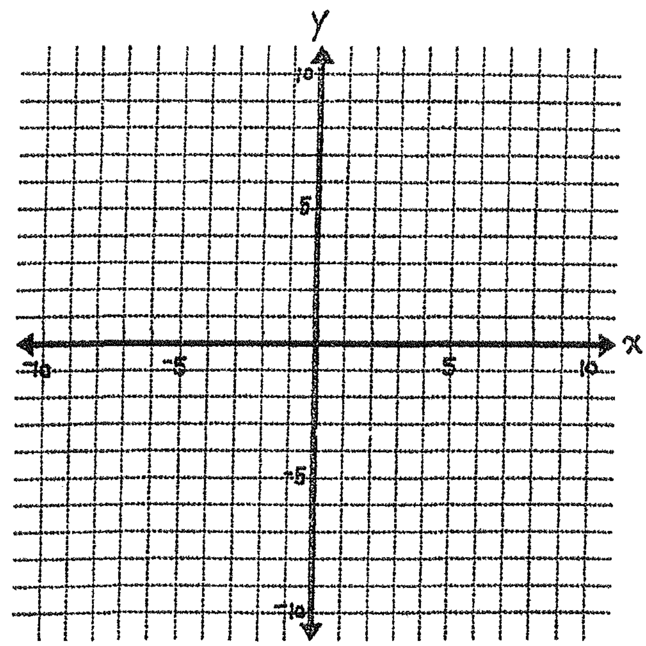
1. Sketch the rays of the angle 30° in the standard position of a coordinate system.

2. Sketch the angle 135° (as in question 1).

3. Sketch the angle $\frac{10\pi}{6}$ (as in question 1).

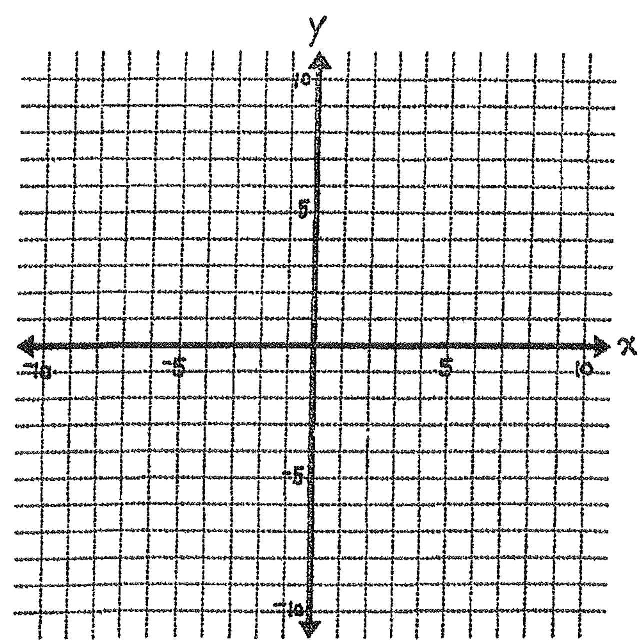
Extra sketching space above

4. Sketch a circle of radius 8 units in the coordinate system, centered at (0, 0).



Circumference = Area =

5. Sketch the sector with radius 6 units, centered at (0, 0) with central angle 60°.



Arc Length = Area =
Total Perimeter=
Values of (x, y) at top corner=

N/D.

NAME: ______________

DIVIDING FRACTIONS AND SQUARE ROOTS

1. Rule: $\dfrac{\frac{a}{b}}{\frac{c}{d}} = \left(\dfrac{a}{b}\right)\left(\dfrac{d}{c}\right) = \dfrac{ad}{bc}$. Example: $\dfrac{\left(\frac{3}{5}\right)}{\left(\frac{7}{11}\right)} = \dfrac{3}{5}\cdot\dfrac{11}{7} = \dfrac{33}{35}$.

a) $\dfrac{\frac{4}{3}}{\frac{6}{8}} =$

b) $\dfrac{2}{\frac{11}{17}} =$

c) $\dfrac{1}{\left(\frac{1}{2}\right)} =$

d) $\dfrac{1}{\left(\frac{4}{3}\right)} =$

2. Rule: Simplify $\dfrac{1}{\sqrt{a}}$ by multiplying by $\dfrac{\sqrt{a}}{\sqrt{a}}$, $\dfrac{1}{\sqrt{a}} = \dfrac{1\cdot\sqrt{a}}{\sqrt{a}\cdot\sqrt{a}} = \dfrac{\sqrt{a}}{a}$.

Example: $\dfrac{1}{\sqrt{11}} = \dfrac{1\cdot\sqrt{11}}{\sqrt{11}\cdot\sqrt{11}} = \dfrac{\sqrt{11}}{11}$.

a) $\dfrac{1}{\sqrt{2}} =$

b) $\dfrac{1}{\sqrt{3}} =$

c) $\dfrac{2}{\sqrt{3}} =$

d) $\dfrac{1}{\left(\frac{2}{\sqrt{3}}\right)} =$

e) $\sqrt{\dfrac{1}{10}} =$

f) $\sqrt{\dfrac{3}{4}} =$

g) $\sqrt{\dfrac{4}{3}} =$

h) $\dfrac{\frac{15}{17}}{\sqrt{\frac{15}{17}}} =$

N40.

Worksheet 6.3: The Unit Circle

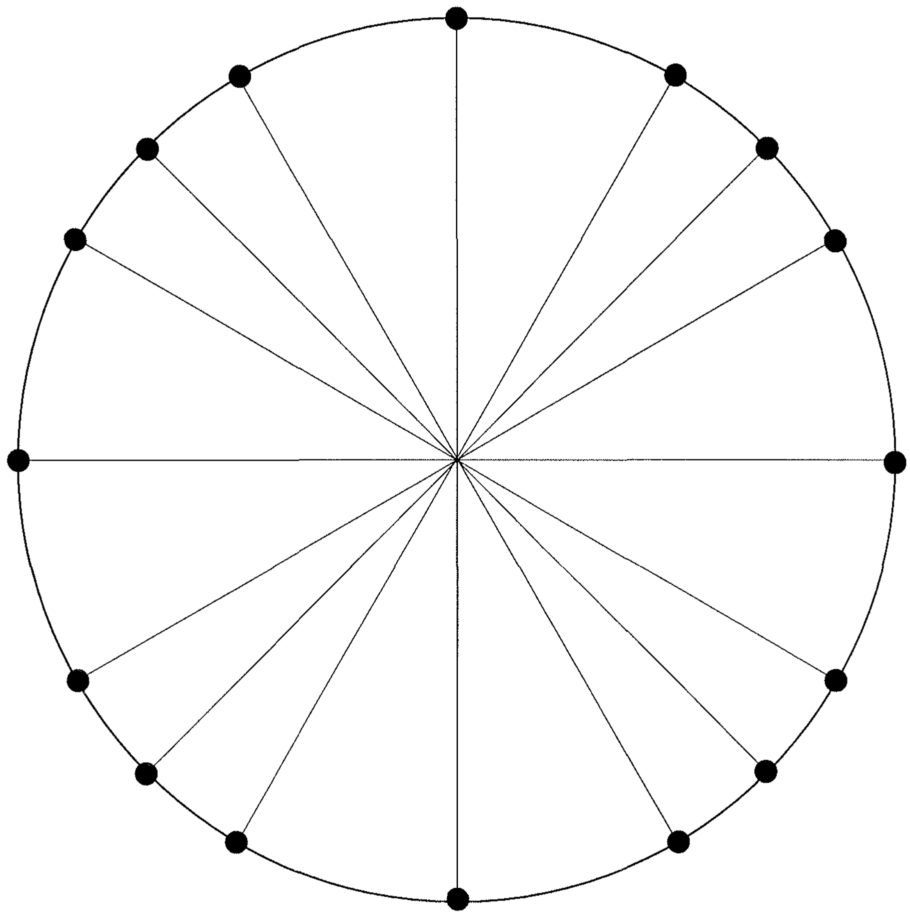


Figure 11

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
θ	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$
$\cos\theta$									
$\sin\theta$									
θ	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π	
θ	$210°$	$225°$	$240°$	$270°$	$300°$	$315°$	$330°$	$360°$	
$\cos\theta$									
$\sin\theta$									

NSO.

NAME: ______________

PERIOD: __________

INVERSE TRIG WORKSHEET

GIVE ALL SOLUTIONS t TO THESE QUESTIONS, IF ANY.

1. WHEN IS $y = 9$, GIVEN
$$y = 7\cos(2t) + 3$$
FOR $0 \le t \le \pi$

a, USE ALGEBRA TO FIND EQUATIONS FOR t

b, FIND t VALUES USING CALCULATOR

TWO t VALUES: WHAT IS ALGEBRAIC RELATIONSHIP?

c, USE GRAPHING CALCULATOR TO FIND t VALUES

2. SOLVE ALGEBRAICALLY:

$y = 3$ FOR $y = 2\sin t$. CHECK WITH GRAPH.

3. $y = 157$ FOR
$$y = 160 + 5\sin(\pi t)$$
IN $0 \le t \le$ (ONE PERIOD)

a, FIND PERIOD.

b, USE A GRAPH TO FIND t VALUES.

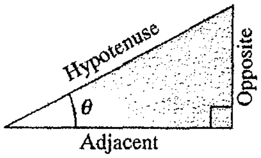
c, WHAT IS ALGEBRAIC RELATIONSHIP BETWEEN t VALUES?

N 75.

TRIGONOMETRY

Definition of the Six Trigonometric Functions

Right triangle definitions, where $0 < \theta < \pi/2$.

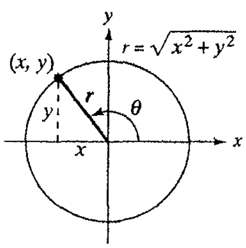


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \quad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \quad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \quad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

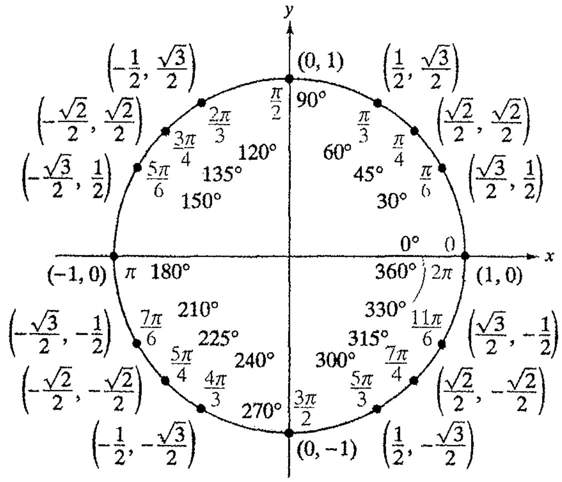
Circular function definitions, where θ is any angle.



$$\sin\theta = \frac{y}{r} \quad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \quad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \quad \cot\theta = \frac{x}{y}$$



Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \quad \sec x = \frac{1}{\cos x} \quad \tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x} \quad \cos x = \frac{1}{\sec x} \quad \cot x = \frac{1}{\tan x}$$

Tangent and Cotangent Identities

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \quad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x \quad \tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \quad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

Reduction Formulas

$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x$$

$$\csc(-x) = -\csc x \quad \tan(-x) = -\tan x$$

$$\sec(-x) = \sec x \quad \cot(-x) = -\cot x$$

Sum and Difference Formulas

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Double-Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2 \sin\left(\frac{u + v}{2}\right)\cos\left(\frac{u - v}{2}\right)$$

$$\sin u - \sin v = 2 \cos\left(\frac{u + v}{2}\right)\sin\left(\frac{u - v}{2}\right)$$

$$\cos u + \cos v = 2 \cos\left(\frac{u + v}{2}\right)\cos\left(\frac{u - v}{2}\right)$$

$$\cos u - \cos v = -2 \sin\left(\frac{u + v}{2}\right)\sin\left(\frac{u - v}{2}\right)$$

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$

N120.

TRIGONOMETRIC FUNCTION GRAPHS:
CALCULATOR EXPLORATIONS WITH TI-83

RESET DEFAULTS

[2nd] [MEM +] 722
- TURN ON EQUATIONS
- MODE DEGREE

[2nd] [QUIT]

[MODE] Degree [2nd] [QUIT (MODE)]

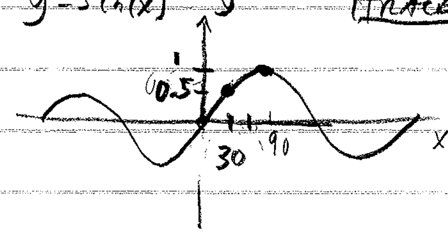
[Y=] $Y_1 = \sin(x)$

USE [ENTER] KEY TO TURN ON/OFF.

[ZOOM] 7: ZTRIG

[TRACE]

$y = \sin(x)$ [TRACE] ◀ ▶



RADIANS

[MODE] Radian

[ZOOM] 7: ZTRIG

[TRACE]

[WINDOW]

XMIN = −352.5
XMAX = 352.5
XSCL = 90

TRY TO CHANGE THESE TO −720, 720 [GRAPH]

YMIN = −4
YMAX = 4
YSCL = 1
XRES = 1

TRY TO CHANGE THESE TO −2, 2. [GRAPH]

[GRAPH]

[2nd] [TABLE (GRAPH)]

[MODE] Degree

[2nd] [TBLSET (WINDOW)]

TblStart = 0

ΔTbl = 15

Auto, Auto.

[2nd] [TABLE (GRAPH)]

Back to TBLSET = 0, 1

PARAMETRIC

[MODE] Par

[Mode] Degree

[Mode] Par

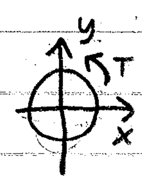
$X_{1T} = \cos(T)$

$Y_{1T} = \sin(T)$

[WINDOW]

TMIN 0
TMAX 720
TSTEP 7.5
XMIN −1.7
XMAX 1.7
XSCL 1
YMIN −1.1
YMAX 1.1
YSCL 1

[TRACE]



[MODE] Func

Back to [MODE] Func

$y_1 = \sin x$ [ZOOM] 7: ZTRIG

A	• $y_2 = 2\sin x$ Vertical stretch
B	• $y_2 = 0.5\sin x$ Shrink
C	• $y_2 = \sin(2x)$ horizontal shrink (period shorter)
D	• $y_2 = \sin(0.5x)$ stretch (period longer)
E	• $y_2 = \sin(x) + 2$ vertical shift up
F	• $y_2 = \sin(x) - 2$ down
G	• $y_2 = \sin(x - 40)$ horizontal shift right
H	• $y_2 = \sin(x + 40)$ left
I	• $y_2 = 1.5\sin(0.5(x - 40)) + 2$
J	• $y_2 = \cos(x)$
K	• $y_2 = \cos(90 - x) - 2$
L	• $y_2 = \tan(x)$

N140

GRAPHING INVERSE RELATIONS IN PARAMETRIC MODE

1/18/08

$\boxed{\text{MODE}}$

RADIANS

PARAMETRIC

$\boxed{\text{Y=}}$

$X_{1T} = \sin(T)$

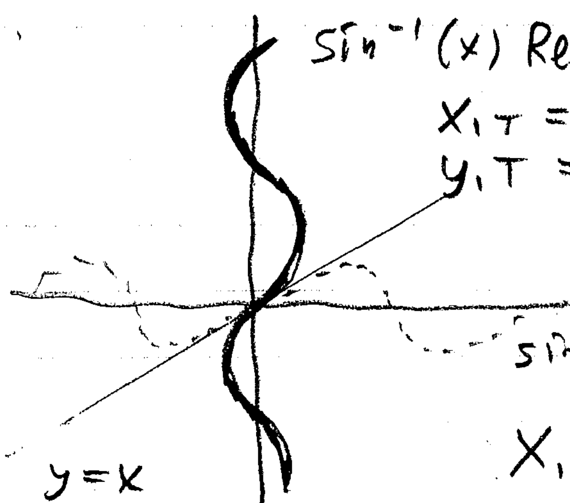
$Y_{1T} = T$

$\boxed{\text{ZOOM}}$ 7: TRIG

$\boxed{\text{WINDOW}}$

$T_{MIN} = -2\pi \approx 6.28$

$\boxed{\text{ZOOM}}$ 5: SQUARE



$\sin^{-1}(x)$ Relation (NOT A FUNCTION)

$X_{1T} = \sin(T)$

$Y_{1T} = T$

sin(x) FUNCTION

$X_{1T} = T$

$Y_{1T} = \sin(T)$

$y = x$

IDENTITY FUNCTION

$X_{1T} = T$

$Y_{1T} = T$

N145.

Sheet#650: Trigonometry Formulas:
Radians, Circular Functions, Master Formula

Radians and Degrees

$$\theta_{\text{deg}} = \frac{180^\circ}{\pi}\theta_{\text{rad}}$$

$$\theta_{\text{rad}} = \frac{\pi}{180^\circ}\theta_{\text{deg}}$$

Arclength

$$s = r\theta_{\text{rad}}$$

Circular Functions
Polar and Cartesian coordinates

$$\sin\theta = \frac{y}{r} \quad \cos\theta = \frac{x}{r} \quad \tan\theta = \frac{y}{x}$$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Master formula

Use either sin(t) or cos(t) as the parent function.

With horizontal shift, h, given:

$$y = \pm A\sin(B(t-h)) + k$$

With phase shift, Bh, given:

$$y = \pm A\sin(Bt - Bh)) + k$$

t = time (or angle θ)
y = height
A = amplitude
B = angular Frequency. $B = \frac{2\pi}{T} = \frac{360^\circ}{T}$
T = period. $T = \frac{2\pi}{B} = \frac{360^\circ}{B}$
h = horizontal Shift
Bh = phase Shift
k = vertical Shift
$y = k$. midline.

f = "regular" frequency (oscillations/second).
$f = 1/T$. $B = 2\pi f$

Name: ____________________________ Block: ___

Sheet 651: Radians, right triangles, circular functions, master formula

$y = \pm A\sin(\ B(t-h)\ ) + k \quad \sin\theta = \frac{y}{r} \quad \cos\theta = \frac{x}{r} \quad \tan\theta = \frac{y}{x}$. Calculator needed.

1. Find the values of each of the three trigonometric functions sine, cosine, and tangent of θ for the following coordinates in the *x*-*y* plane.

   a. (3, 4)

   b. (3, –4)

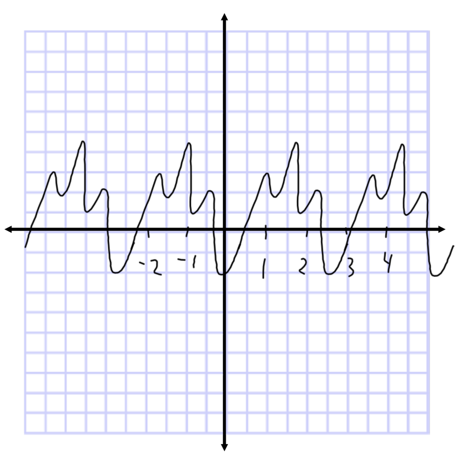
   c. (–5, –7)

   d. (–1, 0)

   e. $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

2. Using the given information, find a good approximation of the period.

   a. The graph, assuming that the horizontal axis is time in units of seconds.



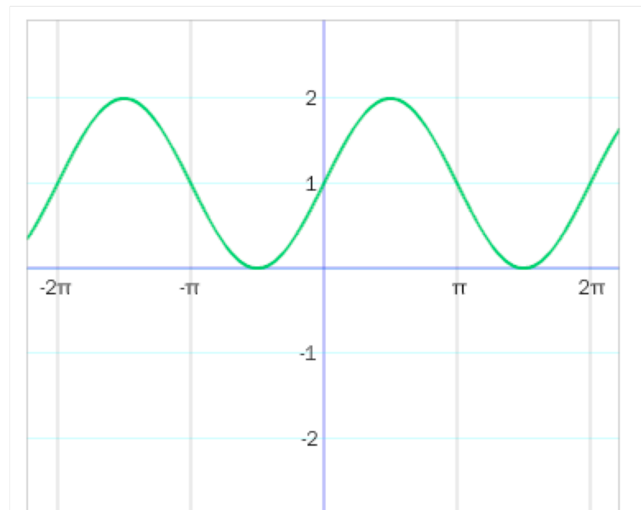
   b. The table

t (minutes)	0	4	8	12	16	20	24	28	32
y (meters)	3	7	9	7	3	7	9	7	3

   c. The position of Earth as it revolves around the Sun.

3. Find the amplitude, angular frequency, period, horizontal shift, phase shift, vertical shift, and the equation of the midline using the given information.

   a. The graph



   b. The formula $y = \pm A\sin(\ B(t-h)\ ) + k$

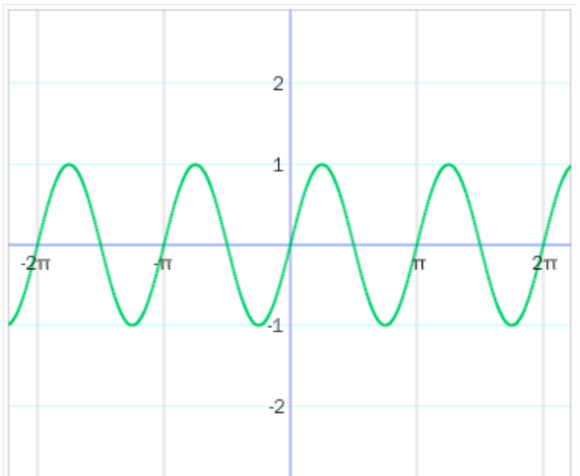
   c. The equation $y = 2\cos(t)$

   d. The equation $y = -5\sin(\ 4(t-3)\ ) + 7$

   e. The equation $y = \sin(\ 8t - 16)$

f. The height, y, of the second hand of a clock with radius of 10 centimeters, starting at the top with $y = 10$ centimeters when time $t = 0$ seconds.

g. The graph



4. Write an equation for a sine function, y versus t, with amplitude of 4, period of $\pi / 4$, and horizontal shift of $\pi / 8$.

5. Convert to degrees: $\pi / 18$

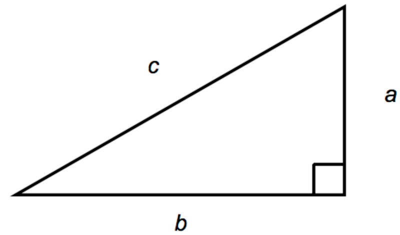
6. Convert to radians: 260°

7. State the reference angle: 260°

8. State the quadrant given this measure: $31\pi / 18$

9. Given a circle, what is the arc length if the radius is 15 meters and the subtended central angle is $\pi / 6$?

10. Given the right triangle shown. The angle A is the angle opposite the side a.



a) Find sin A in terms of the variables given.

b) If $c = 8$ and $b = 3$, find cos A.

c) If $c = 8$ and $b = 3$, find sin A exactly (use integers and square roots as needed but no decimals).

d) If $c = 7$ and $a = 2$, find the measure of the angle A to one decimal.

Name: KEY Block: ___

Sheet 651: Radians, right triangles, circular functions, master formula

$y = \pm A\sin(B(t-h)) + k \quad \sin\theta = \frac{y}{r} \quad \cos\theta = \frac{x}{r} \quad \tan\theta = \frac{y}{x}$. Calculator needed.

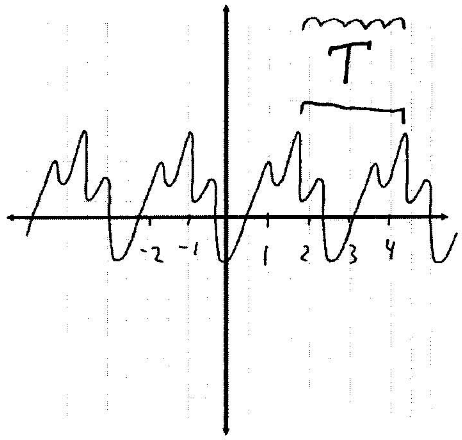
1. Find the values of each of the three trigonometric functions sine, cosine, and tangent of θ for the following coordinates in the *x*-*y* plane.

	r	$\sin\theta$	$\cos\theta$	$\tan\theta$
a. (3, 4)	5	$\frac{4}{5}$	$\frac{3}{5}$	$\frac{4}{3}$
b. (3, –4)	5	$\frac{-4}{5}$	$\frac{3}{5}$	$-\frac{4}{3}$
c. (–5, –7)	$\sqrt{25+49} = \sqrt{74}$	$\frac{-7}{\sqrt{74}}$	$\frac{-5}{\sqrt{74}}$	$\frac{-7}{-5} = \frac{7}{5}$
d. (–1, 0)	1	0	-1	$\frac{0}{-1} = 0$
e. $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$	1	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

2. Using the given information, find a good approximation of the period.

a. The graph, assuming that the horizontal axis is time in units of seconds.

5 BOXES = 2.5 UNITS



$\boxed{T \approx 2.5\text{ sec}}$

b. The table $\quad 24 - 8 = 16 \quad \boxed{T = 16\text{ min}}$

t (minutes)	0	4	8	12	16	20	24	28	32
y (meters)	3	7	9	7	3	7	9	7	3

(DON'T USE THE SEVENS)

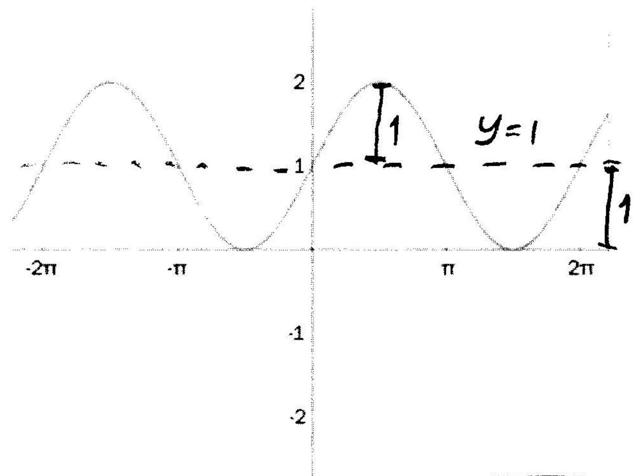
c. The position of Earth as it revolves around the Sun.

$\boxed{T = 1\text{ year}}$

3. Find the amplitude, angular frequency, period, horizontal shift, phase shift, vertical shift, and the equation of the midline using the given information.

a. The graph

$y = \sin(t) + 1$



AMPLITUDE = 1 VERT. SHIFT = 1
ANG. FREQ = 1 MIDLINE: y = 1
PERIOD = 2π
HORIZ SHIFT = PHASE SHIFT = 0

b. The formula $y = \pm A\sin(B(t-h)) + k$

A = AMPLITUDE K = VERT. SHIFT
B = ANG. FREQ y = K : MIDLINE
T = PERIOD
H = HORIZ. SHIFT
BH = PHASE SHIFT

c. The equation $y = 2\cos(t)$

A = 2 k = 0
B = 1 y = 0 MIDLINE
T = 2π
H = BH = 0

d. The equation $y = -5\sin(4(t-3)) + 7$

A = 5 K = 7
B = 4
T = 2π/4 = π/2 y = 7 MIDLINE
H = 3
BH = 12

e. The equation $y = \sin(8t - 16)$ $= \sin(8(t-2))$

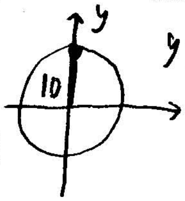
A = 1 K = 0
B = 8 y = 0 MIDLINE.
T = 2π/8 = π/4
BH = 16, so H = 2

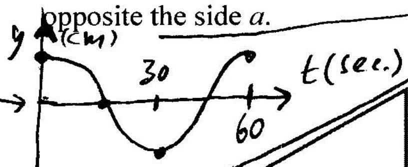
f. The height, *y*, of the second hand of a clock with

(3f.)
radius of 10 centimeters, starting at the top with $y = 10$ centimeters when time $t = 0$ seconds.

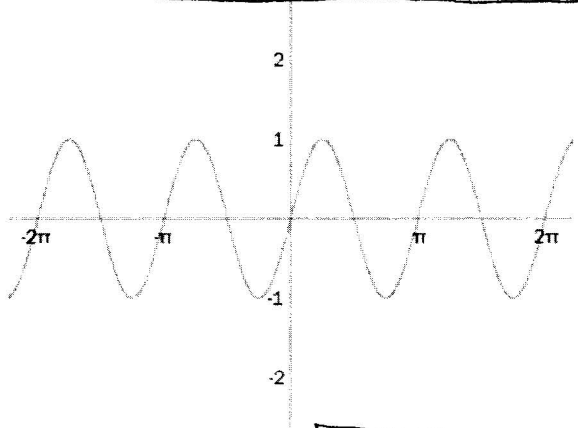
HERE ARE TWO FORMULAS.
*SEE BELOW.

$y = 10 \sin\left(\frac{\pi}{30}(t+15)\right)$
$= 10 \cos(\pi/30\, t)$.





g. The graph



$y = \sin(2t)$

$\boxed{A=1 \quad H=0 \quad B=2 \quad BH=0 \quad T=\pi \quad K=0 \quad y=0 \text{ MIDLINE}}$

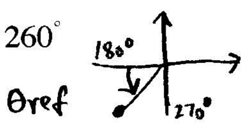
4. Write an equation for a sine function, y versus t, with amplitude of 4, period of $\pi/4$, and horizontal shift of $\pi/8$.

y = 4 sin (8 (t - pi/8))

5. Convert to degrees: $\pi/18 = \frac{180^\circ}{\pi}\left(\frac{\pi}{18}\right) = \boxed{10^\circ}$

6. Convert to radians: $260^\circ = \frac{\pi}{180^\circ}(260^\circ) = \boxed{\frac{13\pi}{9}}$

7. State the reference angle: 260°



$\boxed{+80^\circ}$

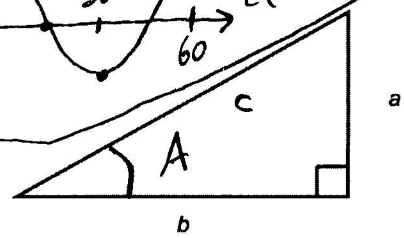
8. State the quadrant given this measure: $31\pi/18$ $\boxed{\text{IV}}$

9. Given a circle, what is the arc length if the radius is 15 meters and the subtended central angle is $\pi/6$?

$s = r\theta = 15\,\text{m} \cdot \frac{\pi}{6} = \boxed{\cancel{2.5\pi\text{m}}}$

2.5 pi meters = 7.9 meters.

10. Given the right triangle shown. The angle A is the angle opposite the side a.



a) Find sin A in terms of the variables given.

$\sin A = \boxed{\frac{a}{c}}$

b) If $c = 8$ and $b = 3$, find cos A.

$\cos A = \boxed{\frac{3}{8}}$

c) If $c = 8$ and $b = 3$, find sin A exactly (use integers and square roots as needed but no decimals).

$a = \sqrt{8^2 - 3^2} = \sqrt{55}$. $\sin A = \boxed{\frac{\sqrt{55}}{8}}$

d) If $c = 7$ and $a = 2$, find the measure of the angle A to one decimal.

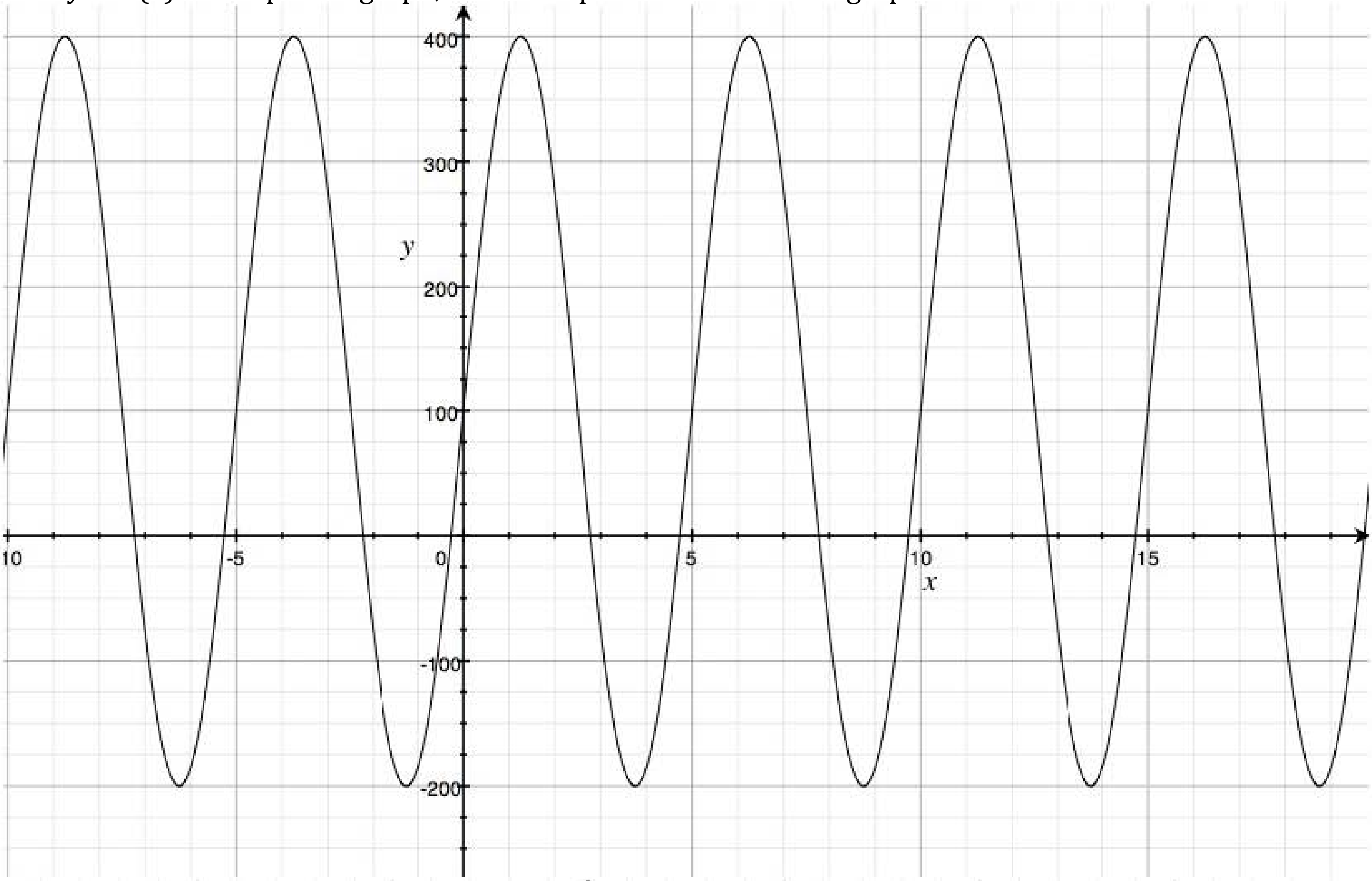
$\sin A = \frac{2}{7}$
$A = \sin^{-1}\left(\frac{2}{7}\right) \approx \boxed{16.6^\circ}$

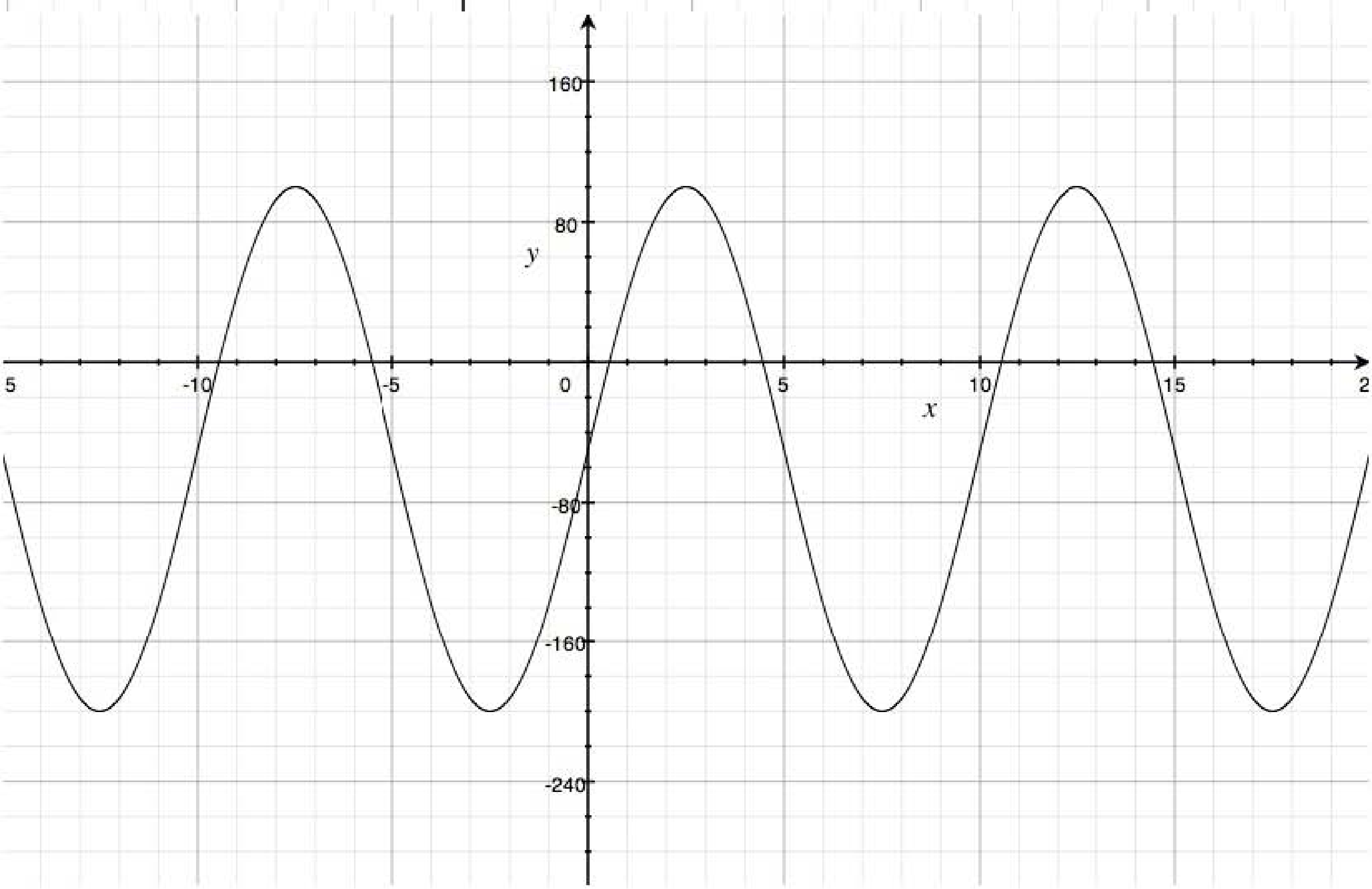
(≈ 0.290 radians)

* QUESTION 3f

	USING SINE	USING COSINE	
A = 10 cm B = π/30 /sec T = 60 sec	H = −15 sec BH = π/2	H = 0 sec BH = 0	K = 0 cm y = 0 cm, MIDLINE

Sheet # 653: Trig functions from graph.
With y=sin(x) as the parent graph, find the equation of each child graph.



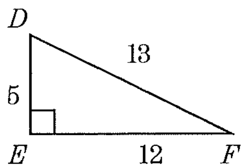


Sheet #673: Trig Functions

Name ________________________________ ________________

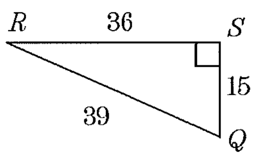
Graphing calculator needed.

1. Find $\tan \angle D$.



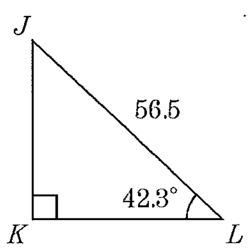
2. Find $\tan \angle F$.

3. Find $\sin \angle R$.



4. If $\sin \angle B = 0.9563$, find $m\angle B$ to the nearest degree.

5. Find JK to the nearest tenth.

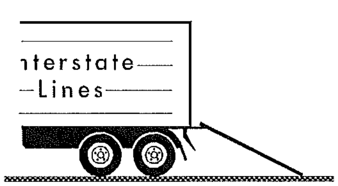


6. Find KL to the nearest tenth.

7. Find $m\angle J$ to the nearest tenth.

8. If $\sin \angle B = \frac{5}{13}$, find $\cos \angle B$.

9. One end of a ramp is raised to the back of a truck 5 feet above the ground. If the length of the ramp is 8 feet, what is the approximate measure of the angle the ramp makes with the ground? Round your answer to the nearest tenth of a degree.



Name the quadrant and reference angle of the given angle.

10. a) 125° b) $28\pi/15$

11. Find 3 functions (not 6). The terminal side of an angle α in standard position passes through the point $(-8, -6)$. Find the six trigonometric functions of α.

12. Complete the following chart. Assume $0 \le \theta < 2\pi$.

radius r	central angle θ	arc length s
7	$\frac{3\pi}{8}$	
4		2π
	$\frac{7\pi}{10}$	$\frac{7\pi}{2}$
1	$\frac{2\pi}{3}$	
2		$\frac{7\pi}{4}$

13. Through how many radians will the hour hand of a clock rotate in

a) 24 hours

b) 6 hours

c) 12 hours

d) 8 hours

Find the amplitude and period of the function.

14. $y = \frac{2}{3}\sin(\theta) + 1$

Find the phase shift and vertical shift of the function.

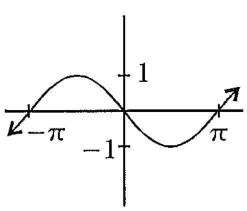
15. $y = 2 + \cos\left(\theta - \frac{\pi}{3}\right)$

Find the amplitude and period of the function.

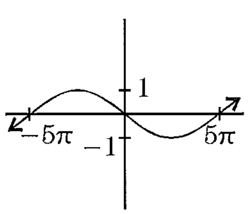
16. $y = \cos\left(\frac{1}{2}x\right) + 8$

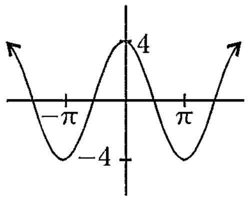
Write the equation of the graph.

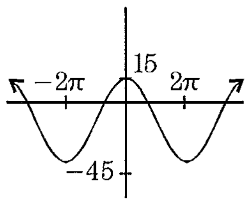
17.

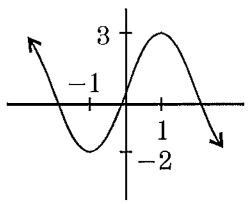


18.



19. 

20. 

21. 

22. Consider the function $y = 5 - 2\cos 3(x + \frac{\pi}{6})$. Without actually graphing the function, write an explanation of how the constants 5, −2, 3, and $\frac{\pi}{6}$ affect the graph, using the graph of $y = \cos x$ as a basis for comparison.

23. Note: frequency $f = 1/T$ and angular frequency $B = 2\pi/T$. The voltage E in an electrical circuit is given by $E = 5\cos 120\pi t$, where t is time measured in seconds.

    a) Find the amplitude and the period of the function.

    b) What is the frequency (number of cycles completed in one second)?

    c) Find E when $t = 0, 0.03, 0.06, 0.09, 0.12$

    d) Graph E, for $0 \le t \le \frac{1}{30}$.

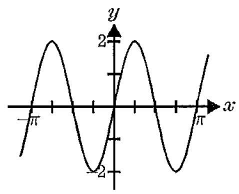
Solve.

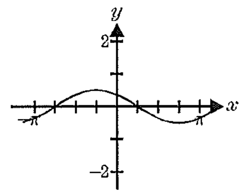
24. $2\cos\theta + 1 = 0$

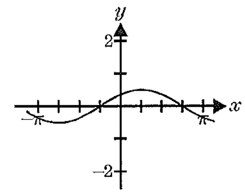
25. $\sin\theta - \frac{\sqrt{3}}{2} = 0$

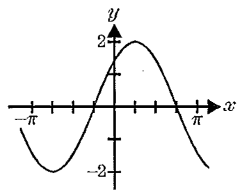
26. If $\cos\theta = -\frac{3}{4}$ and θ lies in quadrant III, find $\sin\theta$.

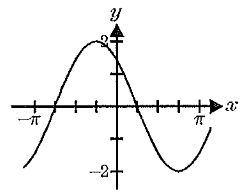
27. Which of the following is the graph of $y = 2\cos(x - \frac{\pi}{4})$?

a)


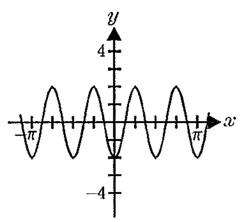
b)


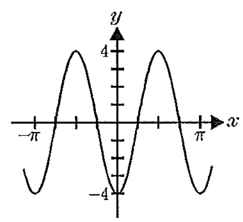
c)


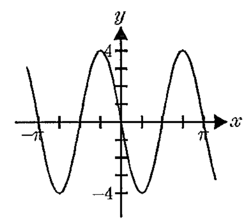
d)


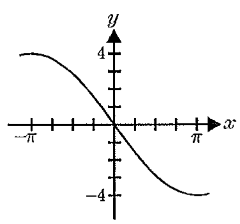
e)


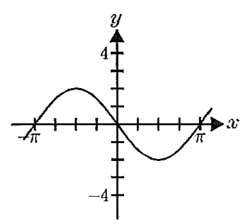
28. Which of the following is the graph of $f(x) = 4\cos(2x - \pi)$?

a)


b)


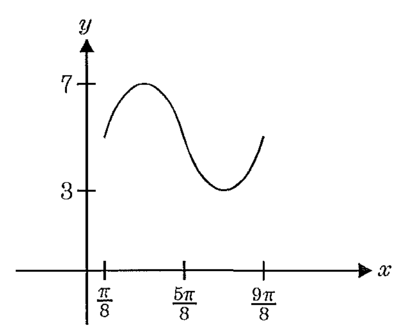
c)


d)


e)


29. This graph illustrates a sine function for one complete cycle. Which of the following is the equation of this graph?

a) $y = 2\sin(2x - \frac{\pi}{4}) + 5$
b) $y = 2\sin(2x + \frac{\pi}{4}) + 5$
c) $y = 3\sin(x - \frac{\pi}{8}) + 5$
d) $y = 3\sin(x + \frac{\pi}{8}) + 5$
e) $y = 2\sin(x - \frac{\pi}{4}) + 5$



30. Solve for x algebraically. Work in radians. Give any one solution. Answer with 3 decimals. Show your work with the appropriate inverse trig function. You may check your work with the calulator.

a) $5 = 3\cos(x) + 4$

b) $9 = 5\sin(4\pi x) + 12$

31. Consider $y = 3\cos(2t) - 1$. Give answers to 4 decimals. Use your graphing calculator in radians.

a) Find the value of y when $t = 2$.

b) Find the first two values of t for which $y = 1$.

32. Consider the equation $y = 3\sin(\pi/2\ t) + 4$. Answer to 4 decimals. Use your graphing calculator in radians.

a) Find the period.

b) Sketch the graph. Label the axes with units and variables.

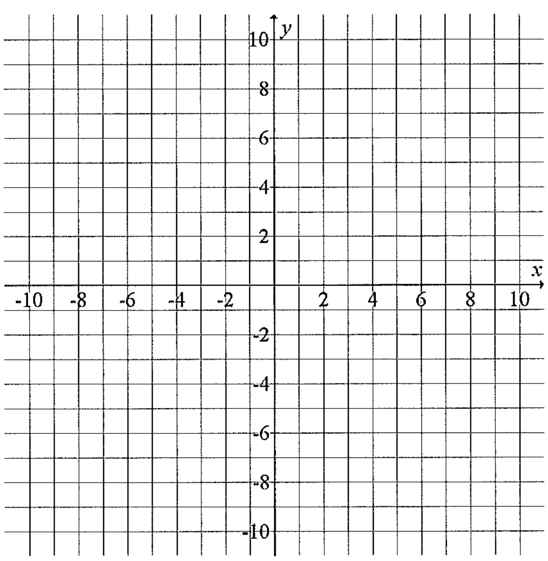
c) Compared to its parent graph, $y = \sin(t)$, how has the graph of the equation above been shifted and/or stretched?

d) Sketch the line $y = 5$.

e) Find the values of t for the first two positive intersections of the line and the curve.

f) How are the two t values related? Give a formula. Explain.

g) What are formulas for the third, fourth, and fifth intersections? (Don't find them, just give the formulas.)



33. Consider a population of deer that varies over the course of a year, from a low in December (time $t = 0$ months) to a high in June ($t = 6$) and back down to a low in December ($t = 12$). One can model the population (y) as a function of time using a sinusoidal function (sine or cosine). In a certain year, there are 2000 animals in December and the population is expected to triple (to 6000) by June.

   a) What is the period?

   b) Write a formula for the number of animals.

   c) Predict the number of animals in March.

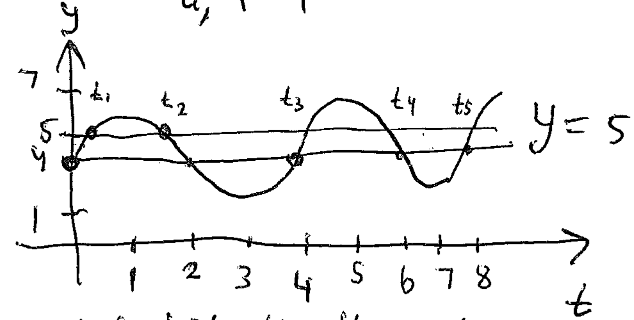
   d) Predict at what times the population will consist of 5000 animals during the course of the year.

Sheet #673: Trig Functions

Answer List

1. $\frac{12}{5}$
2. $\frac{5}{12}$
3. $\frac{5}{13}$
4. 73°
5. 38.0
6. 41.8
7. 47.7°
8. $\frac{12}{13}$
9. 38.7°
10. a, II, 55° b, III $2\pi/15$.
11. $\sin\theta = -\frac{3}{5}$, $\cos\theta = -\frac{4}{5}$, $\tan\theta = \frac{3}{4}$, $\csc\theta = -\frac{5}{3}$, $\sec\theta = -\frac{5}{4}$, $\cot\theta = \frac{4}{3}$
12. →
13. 4π; π; 2π; $\frac{4\pi}{3}$
14. $A = 2/3$. $T = 2\pi$ or 360^0
15. $K=2$ $Bh = \pi/3$
16. $A=1$ $T = 2\pi/\tfrac{1}{2} = 4\pi$
17. $y = -\sin x$
18. $y = -\sin\frac{x}{5}$
19. $y = 4\cos x$
20. $y = 30\cos\left(\frac{x}{2}\right) - 15$
21. $y = \frac{3}{2}\sin\left(\frac{\pi x}{2}\right) + \frac{1}{2}$
22. 5: vert shift up 5 units; −2: makes graph steeper (Vertical stretch, $A=2$.) by a factor of 2 and also reflects graph (MINUS SIGN) across x-axis; 3: Horiz. compression, converts period from 2π to $\frac{2\pi}{3}$; $\frac{\pi}{6}$: Horiz. shift left $\frac{\pi}{6}$ units. (phase shift $= Bh = 3\left(\frac{\pi}{6}\right) = \frac{\pi}{2}$ left.)
23. 5, $\frac{1}{60}$; 60; 5, 1.545, −4.045, −4.045, 1.545
24. $\frac{2\pi}{3}, \frac{4\pi}{3}$
25. $\frac{\pi}{3}, \frac{2\pi}{3}$
26. $-\frac{\sqrt{7}}{4}$
27. d
28. b
29. a
30. a) $\cos^{-1}(1/3) \approx 1.231$. b, $\frac{\sin^{-1}(-3/5)}{4\pi} \approx -0.0512$
31. a, $y = -2.9609$. b, $t_1 = 0.4205$, $t_2 = 2.7211$
32. →
33. a, 12 months b, $y = -2000\cos\left((2\pi/12)t\right) + 4000 = -2000\cos(\pi/6\ t) + 4000$
    c, $y = 4000$. d, $t_1=4$, $t_2=8$. April & August.

32. a, $T=4$



c, Vert. shift = 4. Horiz. shift = 0
Vert. Stretch = 3. $B = \pi/2$ means a Horiz. compression by $\pi/2$

e, $t_1 = 0.2163$
$t_2 = 1.7837$

f, $t_1 + t_2 = 2$ (half the period)

g, $t_3 = t_1 + 4$
$t_4 = t_2 + 4$
$t_5 = t_1 + 8$
Where 4 is the period

12. $\boxed{21\pi/8}$
$2\pi/4 = \boxed{\pi/2}$
$\frac{-\pi/2}{-\pi/10} = \frac{1/2}{1/10} = \frac{10}{2} = \boxed{5}$
$\boxed{\frac{2\pi}{3}}$
$\frac{7\pi}{4}/2 = \boxed{\frac{7\pi}{8}}$

Sheet 675: Rabbit Population

Using the Transformed Sine and Cosine Functions

Sinusoidal functions are used to model oscillating quantities. Starting with $y = A\sin(B(t-h))+k$ or $y = A\cos(B(t-h))+k$, we calculate values of the parameters A, B, h, k.

Example 7 A rabbit population in a national park rises and falls each year. It is at its minimum of 5000 rabbits in January. By July, as the weather warms up and food grows more abundant, the population triples in size. By the following January, the population again falls to 5000 rabbits, completing the annual cycle. Use a trigonometric function to find a possible formula for $R = f(t)$, where R is the size of the rabbit population as a function of t, the number of months since January.

Example 2 Estimate when the rabbit population, R, in Example 7 on page 263 reaches 12,000.

Example 3 As an alternative solution to Example 2, use the inverse cosine to solve the equation:

$$-5000\cos\left(\frac{\pi}{6}t\right) + 10000 = 12000.$$