Sheet 1141: Sequences and Series Review

Sheet#1141: Seq Ser Review

Name $\boxed{\text{KEY}}$ ____________ Per/Sec. ____________

State the first 5 terms of the sequence whose nth term is given by a_n.

1. $a_n = -\frac{3}{n}$

$a_1 = -\frac{3}{1} = \boxed{-3}$

$a_2 = \boxed{-\frac{3}{2}}$

$a_3 = -\frac{3}{3} = \boxed{-1}$

$a_4 = \boxed{-\frac{3}{4}}$

$a_5 = \boxed{-3/5}$

State the next 2 terms of the sequence and give a formula for the nth term.

2. 63, 54, 45, 36, 27, 18, 9.

$-9 \quad -9$

$\boxed{a_n = 63 + (n-1)\cdot(-9)}$

$a_n = 63 - 9n + 9$ $\quad \boxed{a_n = 72 - 9n}$

Find the sum.

3. $\sum_{n=1}^{4} n^4 = 1^4 + 2^4 + 3^4 + 4^4 =$

$= 1 + 16 + 81 + 256 = \boxed{354}$

Express using sigma notation.

4. $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \frac{1}{81} - \frac{1}{243}$ $\quad$ First term = 1.

$\boxed{\sum_{i=1}^{6} \left(\frac{-1}{3}\right)^{i-1}} = \sum_{i=0}^{5} \left(\frac{-1}{3}\right)^{i}$

5. State the 8th term of the geometric sequence $-9, \frac{9}{2}, -\frac{9}{4}, \ldots$.

$a_n = -9\left(-\frac{1}{2}\right)^{n-1}$

$a_8 = -9\left(-\frac{1}{2}\right)^{7} = \boxed{\frac{9}{128}}$

6. The first 3 terms of an arithmetic progression are -15, -17, and -19. Find the 22nd term and the sum of the first 22 terms.

$d = -2$

$a_n = -15 + (n-1)\cdot(-2)$

$a_{22} = -15 + (21)(-2) = \boxed{-57}$

$S_{22} = \frac{1}{2}(-15-57)\cdot 22 = \boxed{-792}$

7. How many terms of the arithmetic series $(-15) + (-8) + (-1) + \cdots$ must be added to give a sum of 801? $\quad$ ANSWER $\boxed{18 \text{ terms}}$

$d = 7$

$a_n = -15 + (n-1)\cdot 7$

$a_n = -15 + 7n - 7$

$a_n = -22 + 7n$

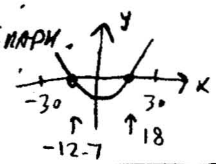
$801 = \frac{1}{2}\underbrace{(-15 + [-22+7n])}_{-37+7n}\cdot n$

$801 = \frac{1}{2}(-37+7n)\cdot n$

$1602 = -37n + 7n^2$ $\quad$ QUADRATIC EQUATION

$7n^2 - 37n - 1602 = 0$

SOLVE BY QUADRATIC FORMULA OR GRAPH.



$X = \frac{37 \pm \sqrt{37^2 + 4\cdot 7\cdot 1602}}{(2\cdot 7)}$

$X = \frac{37 \pm \sqrt{46225}}{14}$

$X = \frac{37 \pm 215}{14} = \begin{cases} 18 \\ -89/7 \approx -12.7 \end{cases}$ $\quad \boxed{N = 18}$

8. Find the sum of the first 6 terms of the geometric series $80 + (-20) + 5 + \cdots$.

$r = -1/4$

$S_6 = 80\left(\frac{1-(-1/4)^6}{1-(-1/4)}\right) = 80\left(\frac{1-\frac{1}{4096}}{1+1/4}\right)$

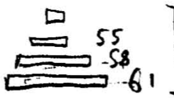
$= 80\,\frac{\left(\frac{4095}{4096}\right)}{\left(\frac{5}{4}\right)} = \frac{80\cdot 4095\cdot 4}{4096\cdot 5} = \boxed{\frac{4095}{64}}$

9. $\frac{1}{2} + (-\frac{1}{4}) + \frac{1}{8} + \cdots$

$r = \frac{-1}{2} \quad a = \frac{1}{2}$

$S = \frac{1}{2}\left(\frac{1}{1-(-1/2)}\right) = \frac{1}{2}\left(\frac{1}{3/2}\right) = \boxed{\frac{1}{3}}$

10. A log pile has 61 logs in the bottom layer, 58 in the second layer, 55 in the third layer, and so on. If there are 639 logs in the pile, how many layers are there?



639 logs

METHOD 2: ADD UNTIL YOU GET TO 639

$61 + 58 + 55 + \cdots + 10 = 639$

#LAYERS = 18

METHOD 1: $639 = \frac{1}{2}(61 + [61 + (N-1)(-3)])\cdot N$

$1278 = (122 - 3N + 3)N$

$3N^2 - 125N + 1278 = 0$ $\quad \boxed{18 \text{ LAYERS}}$

SOLVE: $N = 23.67$ or $N = 18$

11. After the first swing, the path of a pendulum bob is 0.9 as long as long as the previous swing. If the first swing is 50 cm long, how far does the bob travel on the sixth swing? What total distance does the bob travel in those six swings?

$a_n = 50(0.9)^{n-1}$

$a_6 = 50\cdot(0.9)^5 \approx \boxed{29.5\text{ cm}}$

$S_6 = 50\left(\frac{1-0.9^6}{1-0.9}\right) = \boxed{234.3\text{ cm}}$

12. **Formulas**

- Arithmetic sequences and series:

$a_n = a_1 + (n-1)d$ and $S_n = n\left(\frac{a_1 + a_n}{2}\right)$.

- Geometric sequences and series:

$a_n = a_1(r)^{n-1}$ and $S_n = a_1\left(\frac{1-r^n}{1-r}\right)$, and

for $n \to \infty$ with $|r| < 1$, $S = a_1\left(\frac{1}{1-r}\right)$.