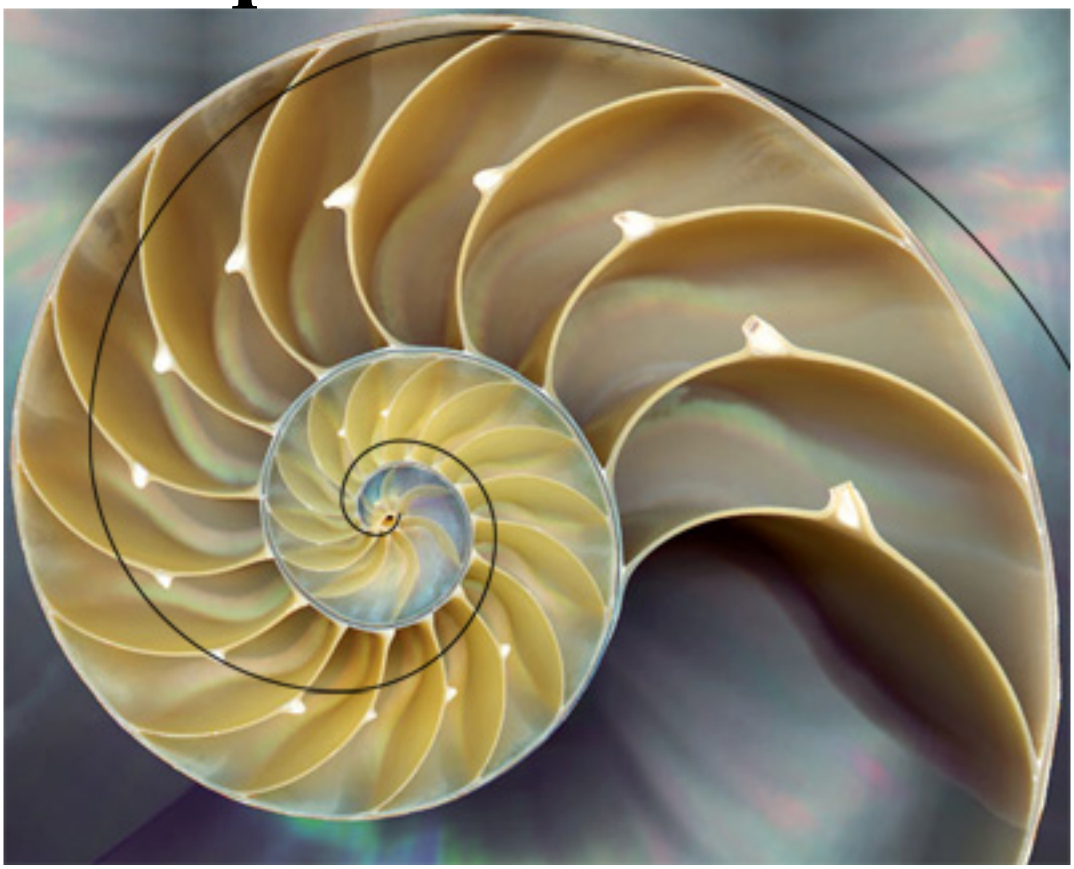
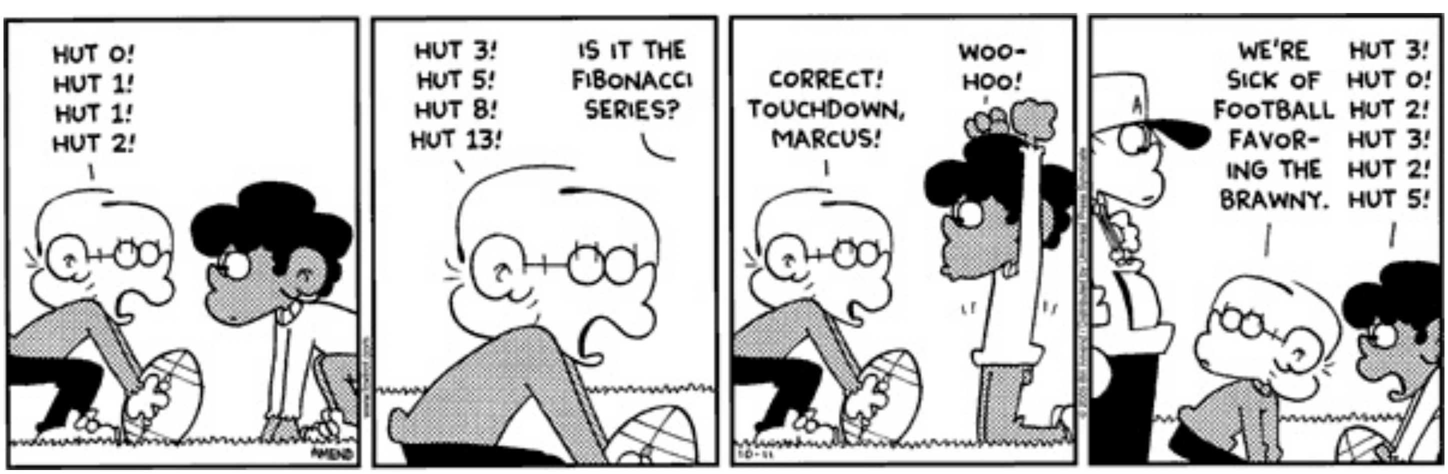
Name: ______________________

# SeqSer Packet

## Sequences and Series






v. 3

LESSON 11.1

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# Practice with Examples

For use with pages 651–657

GOAL **Use and write sequences, and use summation notation to write series and find sums of series**

**VOCABULARY**

A **sequence** could be considered a function whose domain is a set of consecutive integers. If the domain is not specified, it begins at 1.

**Finite sequences** contain a last term, whereas **infinite sequences** continue without stopping.

When the terms of a sequence are added, the resulting expression is a **series.**

**Summation notation,** also called **sigma notation,** is used to write series. For example, in $\sum_{i=1}^{4} 2i$, $i$ is the *index of summation,* 1 is the *lower limit of summation,* and 4 is the *upper limit of summation.*

Chapter 11

## EXAMPLE 1 Writing Terms of Sequences

Write the first five terms of the sequence.

**a.** $a_n = -4n + 3$

**b.** $a_n = (-1)^{n+1}$

**SOLUTION**

Because no domain was specified, begin with $n = 1$.

**a.** $a_1 = -4(1) + 3 = -1$

$a_2 = -4(2) + 3 = -5$

$a_3 = -4(3) + 3 = -9$

$a_4 = -4(4) + 3 = -13$

$a_5 = -4(5) + 3 = -17$

Notice that the terms decrease by 4 each time.

**b.** $a_1 = (-1)^{1+1} = 1$

$a_2 = (-1)^{2+1} = -1$

$a_3 = (-1)^{3+1} = 1$

$a_4 = (-1)^{4+1} = -1$

$a_5 = (-1)^{5+1} = 1$

Notice that the terms in the series alternate from positive to negative.

### Exercises for Example 1

**Write the first five terms of the sequence.**

**1.** $a_n = 5n - 3$

**2.** $a_n = n - 2$

**3.** $a_n = 2^n$

**4.** $a_n = \dfrac{(-1)^n}{n}$

LESSON 11.1 CONTINUED

NAME ______________________ DATE __________

# Practice with Examples

For use with pages 651–657

## EXAMPLE 2 Writing Rules for Sequences

Write the next term in the given sequence. Then write a rule for the $n$th term.

$1, \frac{4}{3}, \frac{6}{4}, \frac{8}{5}, \ldots$

**SOLUTION**

Notice that beyond the first term, in each term the numerator is a multiple of 2 and the denominator increases by 1.

You can rewrite the terms as $\frac{2}{2}, \frac{4}{3}, \frac{6}{4}, \frac{8}{5}, \ldots$.

The next term is $a_5 = \frac{2 \cdot 5}{5+1} = \frac{10}{6}$. A rule for the $n$th term is $a_n = \frac{2n}{n+1}$.

### Exercises for Example 2

**Write the next term in the sequence. Then write a rule for the $n$th term.**

**5.** $3, 6, 9, 12, \ldots$

**6.** $\frac{1}{1}, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots$

**7.** $-4, -3, -2, -1, \ldots$

**8.** $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots$

Chapter 11

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# Practice with Examples

For use with pages 651–657



## EXAMPLE 3 Writing Series with Summation Notation

Write the series with summation notation.

**a.** $1 + 4 + 9 + 16 + \cdots$

**b.** $-2 + 4 - 6 + \cdots + 20$

### SOLUTION

**a.** Notice that the first term is $1^2$, the second is $2^2$, the third is $3^2$, and the fourth is $4^2$. So, the terms of the series can be written as:

$$a_i = i^2 \text{ where } i = 1, 2, 3, 4, \cdots$$

The summation notation for the series is $\sum_{i=1}^{\infty} i^2$.

**b.** Notice that for each terms the signs are alternating negative and positive and the values are multiples of 2. So, the terms of the series can be written as:

$$a_i = (-1)^i 2i \text{ where } i = 1, 2, 3, \ldots, 10.$$

The summation notation for the series is $\sum_{i=1}^{10} (-1)^i 2i$.

### Exercises for Example 3

**Write the series with summation notation.**

**9.** $5 + 6 + 7 + \cdots + 12$

**10.** $-3 - 6 - 9 - \ldots$

**11.** $\frac{1}{3} + \frac{2}{4} + \frac{3}{5} + \cdots + \frac{12}{14}$

**12.** $3 + 5 + 7 + \ldots$



S3

LESSON 11.2

NAME ______________________ DATE ____________

# Practice with Examples

**For use with pages 659–665**

GOAL **Write rules for arithmetic sequences and find sums of arithmetic series**

**VOCABULARY**

In an **arithmetic sequence,** the difference between consecutive terms is constant, and this constant difference is called the **common difference,** denoted by $d$. The $n$th term of an arithmetic sequence with first term $a_1$ and common difference $d$ is given by:

$$a_n = a_1 + (n - 1)d. \quad = nd + (a_1 - d) = nd + a_0$$

The expression formed by adding the terms of an arithmetic sequence is called an **arithmetic series.**

The sum of the first $n$ terms of an arithmetic series is:

$$S_n = n\left(\frac{a_1 + a_n}{2}\right).$$

In words, $S_n$ is the mean of the first and $n$th terms, multiplied by the number of terms.

## EXAMPLE 1 *Identifying Arithmetic Sequences*

Decide whether each sequence is arithmetic.

**a.** 2, 11, 20, 29, 38, . . .

**b.** 2, 4, 7, 11, 16, . . .

**SOLUTION**

To decide whether a sequence is arithmetic, find the difference of consecutive terms. If the differences are constant, then the sequence is arithmetic.

**a.** $a_2 - a_1 = 11 - 2 = 9$

$a_3 - a_2 = 20 - 11 = 9$

$a_4 - a_3 = 29 - 20 = 9$

$a_5 - a_4 = 38 - 29 = 9$

Because each difference is 9, the sequence is arithmetic.

**b.** $a_2 - a_1 = 4 - 2 = 2$

$a_3 - a_2 = 7 - 4 = 3$

$a_4 - a_3 = 11 - 7 = 4$

$a_5 - a_4 = 16 - 11 = 5$

Because the differences are not constant, the sequence is not arithmetic.

### *Exercises for Example 1*

**Decide whether the sequence is arithmetic.**

**1.** 12, 7, 2, −3, −8, . . .

**2.** 4, 8, 16, 22, 32, . . .

**3.** 1, 2, 4, 6, 8, . . .

**4.** 19, 23, 27, 31, 35, . . .

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# Practice with Examples

For use with pages 659–665



## EXAMPLE 2 Writing a Rule for the nth Term

Write a rule for the $n$th term of the sequence 2, 11, 20, 29, 38, . . . . Then find $a_{15}$.

### Solution

The sequence is arithmetic because each difference is 9. So, you can use the rule for the $n$th term:

| | |
|---|---|
| $a_n = a_1 + (n - 1)d$ | Write general rule. |
| $= 2 + (n - 1)9$ | Substitute 2 for $a_1$ and 9 for $d$. |
| $= 2 + 9n - 9$ | Use distributive property. |
| $= 9n - 7$ | Simplify. |

The 15th term is $a_{15} = 9(15) - 7 = 135 - 7 = 128$.

### Exercises for Example 2

**Write a rule for the *n*th term of the sequence and find $a_{15}$.**

**5.** 12, 7, 2, −3, −8, . . .

**6.** 19, 23, 27, 31, 35, . . .

**7.** 5, 7, 9, 11, 13, . . .

**8.** 4, 3, 2, 1, 0, . . .

**9.** 12, 6, 0, −6, −12, . . .

**10.** 5, 8, 11, 14, 17, . . .

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# Practice with Examples

For use with pages 659–665

## EXAMPLE 3 Finding a Sum

Find the sum of the first 50 terms of the arithmetic series
$3 + 6 + 9 + 12 + 15 + \cdots$.

**SOLUTION**

Begin by finding the common difference, $d = 6 - 3 = 3$. Then, find a formula for the $n$th term.

| | |
|---|---|
| $a_n = a_1 + (n - 1)d$ | Write rule for the $n$th term. |
| $= 3 + (n - 1)(3)$ | Substitute 3 for $a_1$ and 3 for $d$. |
| $= 3n$ | Simplify. |

The 50th term is $a_{50} = 3(50) = 150$. So, the sum of the first 50 terms is:

| | |
|---|---|
| $S_{50} = 50\left(\frac{a_1 + a_{50}}{2}\right)$ | Write rule for $S_{50}$. |
| $= 50\left(\frac{3 + 150}{2}\right)$ | Substitute 3 for $a_1$ and 150 for $a_{50}$. |
| $= 3825$ | Simplify. The sum of the first 50 terms is 3825. |

### Exercises for Example 3

**Find the sum of the first *n* terms of the arithmetic series.**

**11.** $2 + 3 + 4 + 5 + \cdots; n = 19$

**12.** $25 + 35 + 45 + 55 + \cdots; n = 50$

LESSON
11.3

NAME ______________________ DATE __________

# Practice with Examples

For use with pages 666–673

GOAL **Write rules for geometric sequences and find sums of geometric series**

**VOCABULARY**

In a **geometric sequence,** the ratio of any term to the previous term is constant, and this constant ratio is called the **common ratio,** denoted by $r$. The $n$th term of a geometric sequence with first term $a_1$ and common ratio $r$ is given by:

$$a_n = a_1r^{n-1}.$$

The expression formed by adding the terms of a geometric sequence is called a **geometric series**. The sum $S_n$ of the first $n$ terms of a geometric series with common ratio $r \neq 1$ is:

$$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right).$$

Chapter 11

EXAMPLE 1

## *Writing a Rule for the nth Term*

Write a rule for the $n$th term of the sequence 6, 24, 96, 384, . . . . Then find $a_7$.

**SOLUTION**

The sequence is geometric because the common ratio $r = \frac{24}{6} = 4$. So, a rule for the $n$th term is:

$$\begin{aligned} a_n &= a_1r^{n-1} \\ &= 6(4)^{n-1} \end{aligned}$$

The 7th term is $a_7 = 6(4)^{7-1} = 6(4)^6 = 24{,}576$.

### *Exercises for Example 1*

**Write a rule for the $n$th term of the geometric sequence.**

**1.** 4, 12, 36, 108, 324, . . .

**2.** 1, 6, 36, 216, 1296, . . .

**3.** 2, −6, 18, −54, 162, . . .

**4.** 7, 14, 28, 56, 128, . . .

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# Practice with Examples

For use with pages 666–673

## EXAMPLE 2 Finding the nth Term Given a Term and the Common Ratio

One term of a geometric sequence is $a_4 = 64$. The common ratio is $r = 4$. Write a rule for the $n$th term.

### SOLUTION

Begin by finding the first term as follows.

| | |
|---|---|
| $a_n = a_1 r^{n-1}$ | Write general rule. |
| $a_4 = a_1 r^{4-1}$ | Substitute 4 for $n$. |
| $64 = a_1(4)^3$ | Substitute 64 for $a_4$ and 4 for $r$. |
| $1 = a_1$ | Solve for $a_1$. |

So, the rule for the $n$th term is:

| | |
|---|---|
| $a_n = a_1 r^{n-1}$ | Write general rule. |
| $= 1(4)^{n-1}$ | Substitute 1 for $a_1$ and 4 for $r$. |

### Exercises for Example 2

**Write a rule for the $n$th term of the geometric sequence.**

**5.** $a_1 = 2, r = -\frac{1}{3}$

**6.** $a_5 = 3, r = -1$

**7.** $a_3 = -12, r = -2$

LESSON 11.3 CONTINUED

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# Practice with Examples

For use with pages 666–673

Chapter 11

## EXAMPLE 3 Finding a Sum

Find the sum of the first ten terms of the geometric series $-3 + 6 - 12 + 24 - \dots$.

### SOLUTION

Begin by finding the common ratio, $r = \dfrac{6}{-3} = -2$. So, the sum of the first 10 terms is:

$$S_{10} = a_1\left(\frac{1 - r^{10}}{1 - r}\right) \quad \text{Write rule for } S_{10}.$$

$$= -3\left(\frac{1 - (-2)^{10}}{1 - (-2)}\right) \quad \text{Substitute } -3 \text{ for } a_1 \text{ and } -2 \text{ for } r.$$

$$= -3\left(-\frac{1023}{3}\right) \quad \text{Simplify.}$$

$$= 1023 \quad \text{Simplify.}$$

The sum of the first 10 terms is 1028.

### Exercises for Example 3

**Find the sum of the first *n* terms of the geometric series.**

**8.** $2 + 8 + 32 + 128 + \cdots ; n = 12$

**9.** $1 + (-6) + 36 + (-216) + \cdots ; n = 9$

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# Practice with Examples

For use with pages 675–680

**GOAL** Find sums of infinite geometric series

**VOCABULARY**

The **sum of an infinite geometric series** with first term $a_1$ and common ratio $r$ is given by

$$S = \frac{a_1}{1 - r}$$

provided $|r| < 1$. If $|r| \geq 1$, the series has no sum.

Chapter 11

## EXAMPLE 1 *Finding Sums of Infinite Geometric Series*

Find the sum of the infinite geometric series, if it has one.

**a.** $\sum_{n=1}^{\infty} 4\left(-\frac{1}{3}\right)^{n-1}$

**b.** $\sum_{n=0}^{\infty} 2\left(\frac{5}{4}\right)^{n}$

**SOLUTION**

**a.** Begin by checking to see if the infinite geometric series has a sum. Because $|r| = \left|-\frac{1}{3}\right| = \frac{1}{3} < 1$, the series has a sum. For this series, $a_1 = 4\left(-\frac{1}{3}\right)^{1-1} = 4\left(-\frac{1}{3}\right)^0 = 4(1) = 4$ and $r = -\frac{1}{3}$. The sum is as follows:

$$S = \frac{a_1}{1 - r} = \frac{4}{1 - \left(-\frac{1}{3}\right)} = \frac{4}{1 + \frac{1}{3}} = \frac{4}{\frac{4}{3}} = 4 \cdot \frac{3}{4} = 3.$$

**b.** Because $|r| = \left|\frac{5}{4}\right| = \frac{5}{4} > 1$, you can conclude that the series has no sum.

### *Exercises for Example 1*

**Find the sum of the infinite geometric series, if it has one.**

**1.** $\sum_{n=0}^{\infty} 3\left(\frac{1}{2}\right)^{n}$

**2.** $\sum_{n=0}^{\infty} 2\left(-\frac{1}{2}\right)^{n}$

**3.** $\sum_{n=1}^{\infty} (0.9)^{n-1}$

**4.** $\sum_{n=0}^{\infty} 3\left(\frac{4}{3}\right)^{n}$

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# Practice with Examples

For use with pages 675–680



## EXAMPLE 2 Finding the Common Ratio

An infinite geometric series with first term $a_1 = 7$ has a sum of 4. What is the common ratio of the series?

**SOLUTION**

| | |
|---|---|
| $S = \frac{a_1}{1 - r}$ | Write rule for sum. |
| $4 = \frac{7}{1 - r}$ | Substitute 4 for $S$ and 7 for $a_1$. |
| $4(1 - r) = 7$ | Multiply each side by $1 - r$. |
| $1 - r = \frac{7}{4}$ | Divide each side by 4. |
| $-r = \frac{3}{4}$ | Subtract 1 from each side. |
| $r = -\frac{3}{4}$ | Solve for $r$. |

### Exercises for Example 2

**Find the common ratio of the infinite geometric series with the given sum and first term.**

**5.** $S = 30, a_1 = 9$

**6.** $S = 3, a_1 = 4$

**7.** $S = 2, a_1 = 1$

**8.** $S = 100, a_1 = 20$

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# Practice with Examples

For use with pages 675–680

## EXAMPLE 3 Writing a Repeating Decimal as a Fraction

Write 0.888 . . . as a fraction.

**SOLUTION**

Notice that only one digit, 8, is repeated in the pattern. So, $r = 0.1$.

$$0.888\ldots = 8(0.1) + 8(0.1)^2 + 8(0.1)^3 + \cdots$$

$S = \dfrac{a_1}{1 - r}$ Write rule for sum.

$= \dfrac{8(0.1)}{1 - 0.1}$ Substitute for $a_1$ and $r$.

$= \dfrac{0.8}{0.9}$ Simplify.

$= \dfrac{8}{9}$ Write as a quotient of integers.

The repeating decimal 0.888 . . . is $\frac{8}{9}$ as a fraction.

### Exercises for Example 3

**Write the repeating decimal as a fraction.**

**9.** 0.111 . . .

**10.** 0.333 . . .

**11.** 0.7171 . . .

**12.** 0.2727 . . .

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# Practice with Examples

For use with pages 681–687

**GOAL** Evaluate and write recursive rules for sequences

**VOCABULARY**

A **recursive rule** gives the beginning term or terms of a sequence and then a recursive equation that tells how $a_n$ is related to one or more preceding terms. The expression $n!$ is read "***n* factorial**" and represents the product of all integers from 1 to $n$. For example, $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$.



## EXAMPLE 1 *Evaluating Recursive Rules*

Write the first five terms of the sequence.

**a.** $a_1 = 1, a_n = a_{n-1} + 3$

**b.** $a_0 = 1, a_n = a_{n-1} + 2n$

**SOLUTION**

**a.**

$a_1 = 1$

$a_2 = a_{2-1} + 3 = a_1 + 3 = 1 + 3 = 4$

$a_3 = a_{3-1} + 3 = a_2 + 3 = 4 + 3 = 7$

$a_4 = a_{4-1} + 3 = a_3 + 3 = 7 + 3 = 10$

$a_5 = a_{5-1} + 3 = a_4 + 3 = 10 + 3 = 13$

**b.**

$a_0 = 1$

$a_1 = a_{1-1} + 2 \cdot 1 = a_0 + 2 = 1 + 2 = 3$

$a_2 = a_{2-1} + 2 \cdot 2 = a_1 + 4 = 3 + 4 = 7$

$a_3 = a_{3-1} + 2 \cdot 3 = a_2 + 6 = 7 + 6 = 13$

$a_4 = a_{4-1} + 2 \cdot 4 = a_3 + 8 = 13 + 8 = 21$

### *Exercises for Example 1*

**Find the first five terms of the sequence.**

**1.** $a_1 = 10, a_n = a_{n-1} - 2$

**2.** $a_1 = 2, a_n = a_{n-1} + n - 1$

**3.** $a_1 = 3, a_n = a_{n-1} + 2$

**4.** $a_1 = 1, a_n = 2a_{n-1}$

NAME ______________________ DATE __________

# Practice with Examples

For use with pages 681–687

## EXAMPLE 2 Writing a Recursive Rule for an Arithmetic Sequence

Write a recursive rule for the arithmetic sequence with $a_1 = 5$ and $d = -3$.

**SOLUTION**

Use the fact that you can obtain $a_n$ in an arithmetic sequence by adding the common difference to the previous term.

$a_n = a_{n-1} + d$ General recursive rule for $a_n$.

$= a_{n-1} - 3$ Substitute $-3$ for $d$.

A recursive rule for the sequence is $a_1 = 5$, $a_n = a_{n-1} - 3$.

### *Exercises for Example 2*

**Write a recursive rule for the arithmetic sequence.**

**5.** $a_1 = 2$, $d = 4$

**6.** $a_1 = 0$, $d = -2$

**7.** $a_1 = -3$, $d = 1$

**8.** $a_1 = 12$, $d = -4$

## EXAMPLE 3 Writing a Recursive Rule for a Geometric Sequence

Write a recursive rule for a geometric sequence with $a_1 = 2$ and $r = -3$.

**SOLUTION**

Use the fact that you can obtain $a_n$ in a geometric sequence by multiplying the previous term by the constant ratio, $r$.

$a_n = r \cdot a_{n-1}$

$= -3a_{n-1}$

A recursive rule for the sequence is $a_1 = 2$, $a_n = -3a_{n-1}$.

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# Practice with Examples

For use with pages 681–687

### Exercises for Example 3

**Write a recursive rule for the geometric sequence.**

**9.** $a_1 = 3$
$r = 8$

**10.** $a_1 = 10$
$r = \frac{1}{2}$

**11.** $a_1 = -2$
$r = 0.3$

**12.** $a_1 = -1$
$r = -2$



## EXAMPLE 4 Writing a Recursive Rule

Write a recursive rule for the sequence 2, 6, 10, 14, 18, . . . .

**SOLUTION**

Notice that the common difference of the terms is 4. This tells you the sequence is arithmetic. Therefore, each term is obtained by adding 4 to the previous term.

A recursive rule is given by:

$a_1 = 2, a_n = a_{n-1} + 4.$

### Exercises for Example 4

**Write a recursive rule for the sequence.**

**13.** 1, 2, 4, 8, 16, . . .

**14.** 11, 23, 35, 47, 59, . . .

**15.** −12, −8, −4, 0, 4, . . .

**16.** 1, −3, 9, −27, . . .

## Chapter 11

### Lesson 11.1

**1.** 2, 7, 12, 17, 22 **2.** $-1, 0, 1, 2, 3$

**3.** 2, 4, 8, 16, 32 **4.** $-1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, -\frac{1}{5}$

**5.** 15; $a_n = 3n$ **6.** $\frac{1}{9}$; $a_n = \frac{1}{2n-1}$

**7.** 0; $a_n = n - 5$ **8.** $\frac{6}{7}$; $a_n = \frac{n+1}{n+2}$

**9.** $\sum_{i=1}^{8} (i + 4)$ **10.** $\sum_{i=1}^{\infty} -3i$ **11.** $\sum_{i=1}^{12} \frac{i}{i+2}$

**12.** $\sum_{i=1}^{\infty} (2i + 1)$

### Lesson 11.2

**1.** yes **2.** no **3.** no **4.** yes

**5.** $a_n = 17 - 5n; -58$ **6.** $a_n = 4n + 15; 75$

**7.** $a_n = 2n + 3; 33$ **8.** $a_n = 5 - n; -10$

**9.** $a_n = 18 - 6n; -72$ **10.** $a_n = 3n + 2; 47$

**11.** 209 **12.** 13,500

### Lesson 11.3

**1.** $a_n = 4(3)^{n-1}$ **2.** $a_n = 1(6)^{n-1}$

**3.** $a_n = 2(-3)^{n-1}$ **4.** $a_n = 7(2)^{n-1}$

**5.** $a_n = 2\left(-\frac{1}{3}\right)^{n-1}$ **6.** $a_n = 3(-1)^{n-1}$

**7.** $a_n = -3(-2)^{n-1}$ **8.** 11,184,810

**9.** 1,439,671

### Lesson 11.4

**1.** 6 **2.** $\frac{4}{3}$ **3.** 10 **4.** none **5.** $\frac{7}{10}$ **6.** $-\frac{1}{3}$

**7.** $\frac{1}{2}$ **8.** $\frac{4}{5}$ **9.** $\frac{1}{9}$ **10.** $\frac{1}{3}$ **11.** $\frac{71}{99}$ **12.** $\frac{3}{11}$

### Lesson 11.5

**1.** 10, 8, 6, 4, 2 **2.** 2, 3, 5, 8, 12

**3.** 3, 5, 7, 9, 11 **4.** 1, 2, 4, 8, 16

**5.** $a_1 = 2, a_n = a_{n-1} + 4$

**6.** $a_1 = 0, a_n = a_{n-1} - 2$

**7.** $a_1 = -3, a_n = a_{n-1} + 1$

**8.** $a_1 = 12, a_n = a_{n-1} - 4$

**9.** $a_1 = 3, a_n = 8a_{n-1}$

**10.** $a_1 = 10, a_n = \left(\frac{1}{2}\right)a_{n-1}$

**11.** $a_1 = -2, a_n = 0.3a_{n-1}$

**12.** $a_1 = -1, a_n = -2a_{n-1}$

**13.** $a_1 = 1, a_n = 2a_{n-1}$

**14.** $a_1 = 11, a_n = a_{n-1} + 12$

**15.** $a_1 = -12, a_n = a_{n-1} + 4$

**16.** $a_1 = 1, a_n = -3a_{n-1}$

# Sheet 0126: scatterplots, correlation, regression

Both Figures 6.2 (page 332) and 6.3 (page 336) have a clear *direction:* the time between eruptions goes up as the length of the previous eruption increases, and life expectancy generally goes up as GDP increases. We say that Figures 6.2 and 6.3 show a *positive association* between the variables.

## Positive association, negative association

Two variables are **positively associated** when above-average values of one tend to accompany above-average values of the other and below-average values also tend to occur together. The scatterplot slopes upward as we move from left to right.

Two variables are **negatively associated** when above-average values of one tend to accompany below-average values of the other, and vice versa. The scatterplot slopes downward from left to right.

Each of our scatterplots has a distinctive *form.* Figure 6.2 shows two *clusters* of eruptions, and Figure 6.3 shows a *curved relationship.* The *strength* of a relationship in a scatterplot is determined by how closely the points follow a clear form. The relationships in Figures 6.2 and 6.3 are not strong. Eruptions of similar lengths show quite a bit of scatter in times until the next eruption, and nations with similar GDPs can have quite different life expectancies. Here is an example of two variables with a strong negative relationship and a simple form.

### EXAMPLE 6.2 How much natural gas does a household use?

Joan is concerned about the amount of energy she uses to heat her home in the Midwest. She keeps a record of the natural gas she consumes each month over one year's heating season. Because the months are not equally long, she divides each month's consumption by the number of days in the month to get the average number of cubic feet of gas used per day. Demand for heating is strongly influenced by the outside temperature. From local weather records, Joan obtains the average temperature for each month, in degrees Fahrenheit. Here are Joan's data:

| Month: | Oct. | Nov. | Dec. | Jan. | Feb. | Mar. | Apr. | May |
|---|---|---|---|---|---|---|---|---|
| Temperature, $x$: | 49.4 | 38.2 | 27.2 | 28.6 | 29.5 | 46.4 | 49.7 | 57.1 |
| Gas consumed, $y$: | 520 | 610 | 870 | 850 | 880 | 490 | 450 | 250 |



A scatterplot of these data appears in Figure 6.5. Temperature is the explanatory variable $x$ (plotted on the horizontal axis) because outside temperature helps explain gas consumption. The scatterplot shows *a strong negative straight-line association.* The straight-line form is important because it is common and simple. The association is strong because the points lie close to a line. It is negative because as temperature increases, gas consumption goes down because less gas is used for heating.

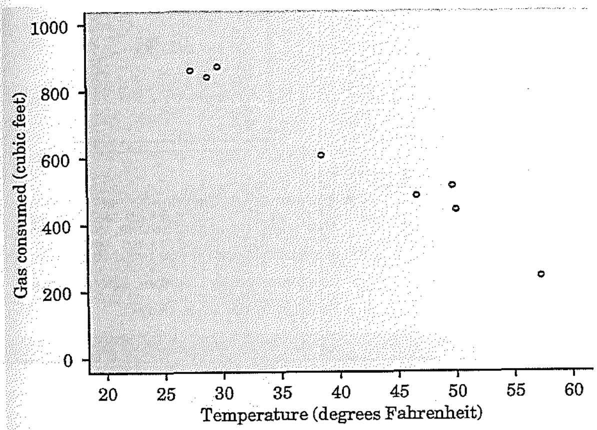
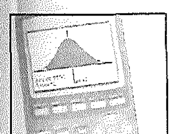



FIGURE 6.5 Home consumption of natural gas versus outdoor temperature.

### CALCULATOR CORNER How to make a scatterplot



In this Calculator Corner, you will make a scatterplot of the natural gas consumption data from Example 6.2.

- Go into your Statistics/List Editor (press STAT then choose `1:Edit...`).

Clear lists $L_1$ and $L_2$ as you did in Activity 6.1.

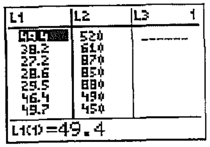
- Enter the temperature values in list $L_1$ and the corresponding gas consumed values in list $L_2$.



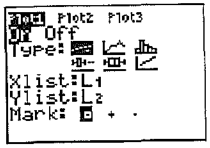
STA 2005
Yates, Starnes, Moore

# Sheet 0126: scatterplots, correlation, regression

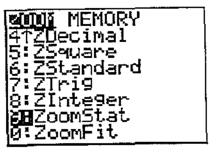
**CALCULATOR CORNER** How to make a scatterplot *(continued)*

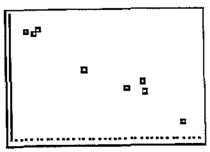



- Go to the statistics plots menu (press [2nd] [Y=]) and select `1:Plot1`. Adjust your settings as shown.

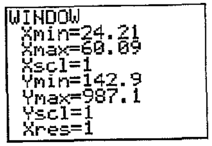



- Make sure that Plot2 and Plot3 are Off, and that all functions are deactivated in your Y= menu. Then use `ZoomStat` (press [ZOOM] and choose `9:ZoomStat`) to produce the graph.

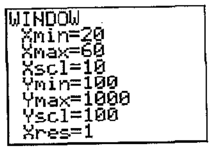


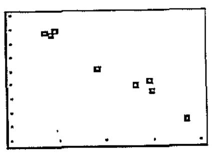
- One disadvantage of using `ZoomStat` is that the calculator chooses the viewing window for you. Press [WINDOW]. Notice the unusual choices for Xmin, Xmax, Ymin, and Ymax.




- You can adjust the window settings to values that seem more reasonable, which can be especially useful if you want to transfer a calculator graph to paper. Change your settings to the ones shown.



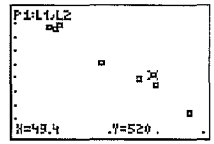

- Press [GRAPH]. Now the tick marks around the edges of the window give you a better idea of each point's location.



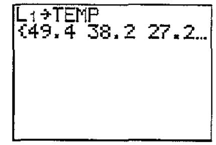


**CALCULATOR CORNER** How to make a scatterplot *(continued)*

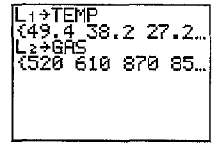
- Press [TRACE] and use your arrow keys to move from one point to another in the scatterplot. Notice that the *x* and *y* coordinates of each point you trace are given at the bottom of the screen.




- Save the data for later use. First, go to the home screen (press [2nd] [MODE] to get there). Then press [2nd] [1] [STO →] [2nd] [ALPHA], type the letters TEMP (the letters are in green above the keys), and press [ENTER]. Compare with the screen shot shown.




- Follow the same procedure to store the values in list $L_2$ to the variable GAS.




## EXERCISES

6.5 Rich states, poor states One measure of a state's prosperity is the median income of its households. Another measure is the mean personal income per person in the state. Figure 6.6 (next page) is a scatterplot of these two variables, both measured in thousands of dollars. Because both variables have the same units, the plot is square with the same scales on both axes.

(a) Explain why you expect a positive association between these variables. Also, explain why you expect household income to be generally higher than income per person.

(b) Nonetheless, the mean income per person in a state can be higher than the median household income. In fact, the District of Columbia has median income $33,433 per household and mean income $38,429 per person. Explain why this can happen.

(c) Describe the overall pattern of the plot, ignoring the outliers.

(d) We have labeled some interesting states on the graph. New York has low median household income relative to its mean individual income. In Utah, median household income is high relative to mean individual income. What facts about a state could make median household income unusually high or low relative to mean individual income?

# Sheet 0126: scatterplots, correlation, regression

1982. The correlation for all 50 states in Figure 6.18 is 0.408. If we leave out Hawaii, the correlation drops to $r = 0.195$. The solid line in the figure is the least-squares line for predicting the annual record from the 24-hour record. If we leave out Hawaii, the least-squares line drops down to the dotted line. This line is nearly flat—there is little relation between yearly and 24-hour record precipitation once we decide to ignore Hawaii.

The usefulness of the regression line for prediction depends on the strength of the association. That is, the usefulness of a regression line depends on the correlation between the variables. It turns out that the square of the correlation is the right measure to use.

## $r^2$ in regression

The **square of the correlation,** $r^2$, is the fraction of the variation in the values of $y$ that is explained by the least-squares regression of $y$ on $x$.

The idea is that when there is a straight-line relationship, some of the variation in $y$ is accounted for by the fact that as $x$ changes it pulls $y$ along with it.

### EXAMPLE 6.8 Using $r^2$

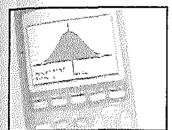
Look again at Figure 6.14 (page 365). There is a lot of variation in the humerus lengths of these 5 fossils, from a low of 41 cm to a high of 84 cm. The scatterplot shows that we can explain almost all of this variation by looking at femur length and at the regression line. As femur length increases, it pulls humerus length up with it along the line. There is very little leftover variation in humerus length, which appears in the scatter of points about the line. Because $r = 0.994$ for these data, $r^2 = (0.994)^2 = 0.988$. So the variation "along the line" as femur length pulls humerus length with it accounts for 98.8% of all the variation in humerus length. The scatter of the points about the line accounts for only the remaining 1.2%. Little leftover scatter says that prediction will be accurate.

Contrast the voting data in Figure 6.15 (page 366). There is still a straight-line relationship between the 1980 and 1984 Democratic votes, but there is also much more scatter of points about the regression line. Here, $r = 0.704$ and so $r^2 = 0.496$. Only about half the observed variation in the 1984 Democratic vote is explained by the straight-line pattern. You would still guess a higher 1984 Democratic vote for a state that was 45% Democratic in 1980 than for a state that was only 30% Democratic in 1980. But lots of variation remains in the 1984 votes of states with the same 1980 vote. That is the other half of the



total variation among the states in 1984. It is due to other factors, such as differences in the main issues in the two elections and the fact that President Reagan's two Democratic opponents came from different parts of the country.

In reporting a regression, it is usual to give $r^2$ as a measure of how successful the regression was in explaining the response. When you see a correlation, square it to get a better feel for the strength of the association. Perfect correlation ($r = -1$ or $r = 1$) means the points lie exactly on a line. Then $r^2 = 1$ and all of the variation in one variable is accounted for by the straight-line relationship with the other variable. If $r = -0.7$ or $r = 0.7$, $r^2 = 0.49$ and about half the variation is accounted for by the straight-line relationship. In the scale, correlation $\pm 0.7$ is about halfway between 0 and $\pm 1$.



### CALCULATOR CORNER Regression and prediction

In this Calculator Corner, you will learn to use your calculator to compute the equation of the least-squares regression line. Then you will discover how to use your equation to make predictions. We will use the temperature and natural gas data from Example 6.2 (page 340).

- Reload the temperature data into list $L_1$ and the natural gas consumed data into list $L_2$. If you saved the lists in the previous Calculator Corner (page 341), you can find them in the LIST menu.
- Make a scatterplot of the data with temperature as the explanatory variable.
- Turn your diagnostics on. This will ensure that the calculator displays both $r$ and $r^2$ along with the linear regression equation. Go into the CATALOG by pressing 2nd 0. Scroll down until your cursor is next to DiagnosticOn. Then press ENTER twice.
- To calculate the equation of the least-squares regression line: from the home screen, press STAT, arrow right to CALC, and choose `8:LinReg(a+bx)`.

```
EDIT CALC TESTS
2↑2-Var Stats
3:Med-Med
4:LinReg(ax+b)
5:QuadReg
6:CubicReg
7:QuartReg
8:LinReg(a+bx)
```

- Then execute the command `LinReg (a+bx) L1,L2,Y1`. (To find $Y_1$, press VARS, then arrow to `Y-VARS`, choose `1:Function` and `1:Y1`.) Compare your result with the screen below.

```
LinReg
 y=a+bx
 a=1425.027417
 b=-19.87187775
 r²=.9656325261
 r=-.9826660298
```

# Sheet 0126: scatterplots, correlation, regression

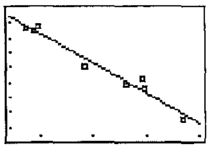
**CALCULATOR CORNER** Regression and prediction *(continued)*

- Let's look at the calculator output. The equation of the least-squares regression line is $y = 1425 - 19.9x$, or in plain language,

natural gas consumed
$$= 1425 - (19.9 \times \text{temperature})$$

Note that the value of the correlation, $r = -0.983$, is displayed as part of the regression output.

- Press GRAPH to display the regression line on your scatterplot. Notice how well the line fits the data. The $r^2$-value displayed earlier, 0.966, tells us that 96.6% of the variation in natural gas consumed is explained by this least-squares regression.



- Let's use the regression line to predict the amount of natural gas Joan will consume in a month with an average temperature of 30° F. From the home screen, execute the command $Y_1(30)$. You should obtain a prediction of 828.9 cubic feet per day. To confirm this value, note that

$$y = 1425 - (19.9)(30) = 828$$

which differs slightly from the calculator result due to rounding.

```
Y1(30)
        828.8710843
```

- Can you interpret the slope and the $y$ intercept of the regression line in this setting? The slope tells us that gas consumption goes down 19.9 cubic feet per day when the average outdoor temperature rises by 1 degree, according to our regression model. The $y$ intercept suggests that if the average outdoor temperature in Joan's area was 0° F, she would use about 1425 cubic feet of natural gas per day.

***Caution:*** Since the data that were used to construct this linear model had $x$-values ranging from about 27 to 57° F, it is risky to use the regression equation to predict gas consumption in a month with an average temperature of 0° F.

The prediction in the Calculator Corner is based on fitting a regression line to the data for eight past months. Joan's house will almost certainly not use exactly 828 cubic feet of gas per day during the next month that has average temperature



30 °F. Because the data points lie close to the line, however, we can be confident that gas consumption in such a month will be quite close to 828 cubic feet per day.

## EXERCISES

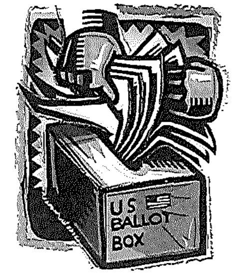
**6.29 Obesity in mothers and daughters** A study found that the correlation between the body mass index (BMI) of young girls and their minutes of physical activity in a day was $r = -0.18$.[15] Why might we expect this correlation to be negative? What percent of the variation in BMI among the girls in the study can be explained by the straight-line relationship with minutes of activity?

**6.30 Correlation and regression** If the correlation between two variables $x$ and $y$ is $r = 0$, there is no straight-line relationship between the variables. It turns out that the correlation is 0 exactly when the slope of the least-squares regression line is 0. Explain why slope 0 means that there is no relationship between $x$ and $y$. Start by drawing a line with slope 0 and explaining why in this situation $x$ has no value for predicting $y$.

**6.31 $r$ versus $r^2$** "When $r = 0.7$, this means that $y$ can be predicted from $x$ for 70% of the individuals in the sample." Is this statement true or false? Would it be true if $r^2 = 0.7$? Explain your answers.

**6.32 Activity 6.2 follow-up**
(a) Use your calculator to find the equation of the least-squares regression line for your data from Activity 6.2 (page 364). Follow the method in the Calculator Corner (page 375). If you saved the lists ARM and HEIT in Activity 6.2, you should use them now.
(b) Interpret the slope and $y$ intercept of the regression line in this setting.
(c) Use your regression equation to predict the height of a student whose measured distance on the arm is 15 inches.

**6.33 Beer and BAC** How much does drinking beer increase the alcohol content of your blood? This question was addressed in an experiment at Ohio State University. Sixteen students volunteered to take part in an experiment. Before the experiment, each of the students blew into a Breathalyzer to show that their




### Did the vote counters cheat?

Republican Bruce Marks was ahead of Democrat William Stinson when the voting machines were tallied in their Pennsylvania election. But Stinson was ahead after absentee ballots were counted by the Democrats who controlled the election board. A court fight followed. The court called in a statistician, who used regression with data from past elections to predict the counts of absentee ballots from the voting-machine results. Marks's lead of 564 votes from the machines predicted that he would get 133 more absentee votes than Stinson. In fact, Stinson got 1025 more absentee votes than Marks. Did the vote counters cheat?

Name: ___________________________

# Sheet #1110: Definitions of Sequence and Series

## Sequence

In mathematics, a **sequence** is an ordered list of objects (or events). Like a set, it contains members (also called *elements* or *terms*), and the number of terms (possibly infinite) is called the *length* of the sequence. Unlike a set, order matters, and exactly the same elements can appear multiple times at different positions in the sequence. A sequence is a discrete function.

For example, (C, R, Y) is a sequence of letters that differs from (Y, C, R), as the ordering matters. Sequences can be *finite*, as in this example, or *infinite*, such as the sequence of all even positive integers (2, 4, 6...). Finite sequences are sometimes known as *strings* or *words* and infinite sequences as *streams*. The empty sequence ( ) is included in most notions of sequence, but may be excluded depending on the context.

## Series

A **series** is the sum of the terms of a sequence. **Finite sequences and series** have defined first and last terms, whereas **infinite sequences and series** continue indefinitely.

### Potential confusion

When talking about series, one can refer either to the sequence $\{ S_N \}$ of the partial sums, or to the *sum of the series*,

$$\sum_{n=0}^{\infty} a_n$$

*i.e.*, the limit of the sequence of partial sums (see the formal definition in the next section) – it is clear which one is meant from context.

*Wikipedia*

| | |
|---|---|
| Sequence: | Ordered list of numbers obeying a rule. |
| | Function whose domain is a set of consecutive integers. |
| Series: | The value of the sum of the terms of a sequence. |

**Sheet 1111: Guess the Sequence, I**

Guess the Terms of the Sequence and Find the Nth Term of the Sequence (#1)

| *Position* | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Term* | | 6 | 7 | 8 | | | | | | | |

Nth Term Sequence Rule:

| *Position* | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Term* | | 5 | 7 | 9 | | | | | | | |

Nth Term Sequence Rule:

| *Position* | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Term* | | -3 | -2 | -1 | | | | | | | |

Nth Term Sequence Rule:

| *Position* | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Term* | | 9 | 11 | 13 | | | | | | | |

Nth Term Sequence Rule:

Guess the Terms of the Sequence and Find the Nth Term of the Sequence (#2)

| *Position* | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Term* | | 9 | | 15 | | 21 | | | | | |

Nth Term Sequence Rule:

| *Position* | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Term* | | -6 | -3 | 0 | | | | | | | |

Nth Term Sequence Rule:

| *Position* | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Term* | | | | -5 | -3 | -1 | | | | | |

Nth Term Sequence Rule:

| *Position* | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Term* | | | | 29 | | 39 | | 49 | | | |

Nth Term Sequence Rule:

Guess the Terms of the Sequence and Find the Nth Term of the Sequence (#3)

**Sheet 1111: Guess the Sequence, I**

| *Position* | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Term* | | | 5 | | 6 | | 7 | | | | |

Nth Term Sequence Rule:

| *Position* | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Term* | | | | | | | | | 9 | | 13 |

Nth Term Sequence Rule:

| *Position* | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Term* | | | | | | 44 | 51 | | | | |

Nth Term Sequence Rule:

| *Position* | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Term* | | -6 | 7 | | | | | | | | |

Nth Term Sequence Rule:

Guess the Terms of the Sequence and Find the Nth Term of the Sequence (#4)

| *Position* | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Term* | | 1 | 4 | 9 | 16 | | | | | | |

Nth Term Sequence Rule:

| *Position* | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Term* | | 5 | 8 | 13 | 20 | | | | | | |

Nth Term Sequence Rule:

| *Position* | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Term* | | 5 | 11 | 21 | 35 | | | | | | |

Nth Term Sequence Rule:

| *Position* | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Term* | | -3 | 3 | 13 | 27 | | | | | | |

Nth Term Sequence Rule:

Name: ____________________________

# Sheet #1122: Arithmetic Sequences and Series

Fred Kral, v.2

**1. Definitions.**

a) Sequence

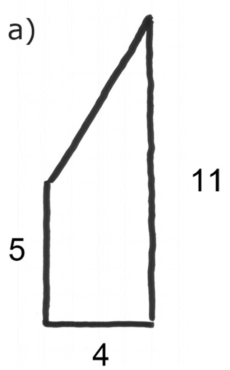
b) Series

c) Arithmetic Sequence

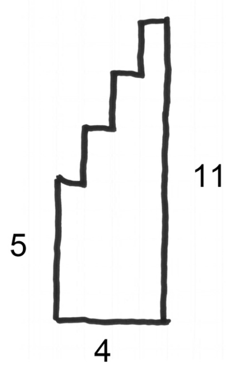
d) Write a formula for the *n*th term of an arithmetic sequence

**2. Find the areas.**

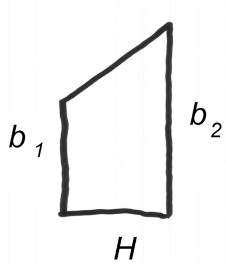
a)



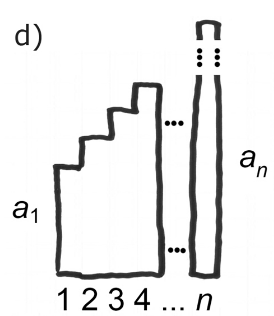

c)




b)




d)




**3. Series formula.**

Write a formula for the sum of the first *n* terms of an arithmetic sequence

**4. Exercises.**

a) Write a formula the *n*th term of the arithmetic sequence 4, 6, 8, ….

b) Find the sum of the first 7 terms of the arithmetic sequence 4, 6, 8, ….

by Keith Hubert

**Problems.**

1. A theater has seating can be arranged so the first row has 10 seats, the second row has 14 seats, the third row has 18 seats and so on. There is sufficient space for a total of 31 rows.

a) How many seats are there in the last row?

b) How many total seats are there?

2. A stack of bricks is composed of horizontal layers. As you go up the stack, each layer has one fewer brick than the layer below. The top layer contains one brick. The bottom layer contains $n$ bricks.

a) Write an equation for the number of bricks in layer $n$.

b) Write an equation for the sum of the number of bricks from the top layer down to the $n$th layer. Write the sum using only one variable, $n$.

c) There are 105 bricks in the stack. Write a quadratic equation whose solution gives the number of layers in the stack.

d) Find the number of layers.

3. Tickets for a certain show were printed bearing numbers from 1 to 100. Odd number tickets were sold by receiving cents equal to three times the number printed on the ticket. Even numbered tickets were sold by receiving cents equal to two times the number printed on the ticket. How many dollars were made in ticket sales?

4. A factory owner repays a loan of $2,088,000 by $20,000 in the first monthly installment and then increases the payment by $1000 in every installment. In how many installments will the loan be paid off?

Sheet #1130:
PRACTICE FOR SEQUENCES

NAME: ____________________

Period: ______

1. 2, 7, 12, 17, 22, ...

a) WRITE AN EXPLICIT RULE FOR $a_n$ (NOT USING $a_{n-1}$).

b) WHAT IS $21^{st}$ ELEMENT ?

2. CONSIDER THE GEOMETRIC SEQUENCE WITH $a_1 = 9$ AND $r = 1/3$

a) WRITE AN EXPLICIT RULE FOR $a_n$.

b) WRITE A RECURSIVE RULE.

c) WHAT IS 4th ELEMENT ?

3. FIND THE FIRST 4 ELEMENTS OF

$$\begin{cases} a_1 = 3 \\ a_{n+1} = a_n + 2 \end{cases}$$

Sheet #1130:

PRACTICE FOR SEQUENCES

NAME: $\boxed{\text{KEY}}$

Period: ____

1. 2, 7, 12, 17, 22, ...

a) WRITE AN EXPLICIT RULE FOR $a_n$ (NOT USING $a_{n-1}$).

$d = 5$ $\boxed{a_n = 2 + (n-1)\cdot 5}$

$= -3 + 5n$

b) WHAT IS $21^{st}$ ELEMENT?

$a_{21} = 2 + (20)\cdot 5 = \boxed{102}$

2. WRITE A $\underbrace{\text{SEQUENCE}}_{}$ WITH RULE FOR A GEOMETRIC

$a_1 = 9$, $r = 1/3$

a) $\boxed{a_n = 9 \cdot \left(\frac{1}{3}\right)^{n-1}} = 27 \cdot \left(\frac{1}{3}\right)^n$

b) RECURSIVE: $\boxed{\begin{cases} a_1 = 9 \\ a_n = a_{n-1} \cdot \frac{1}{3} \end{cases}}$

c) $a_4 = 9 \cdot \left(\frac{1}{3}\right)^3 = \boxed{\frac{1}{3}}$

3. FIND THE FIRST 4 ELEMENTS OF

$a_1 = 3$

$a_{n+1} = a_n + 2$

$a_1 = \boxed{3}$

$a_2 = 3 + 2 = \boxed{5}$

$a_3 = 5 + 2 = \boxed{7}$

$a_4 = 7 + 2 = \boxed{9}$

Name:______________ Period:______

Sheet# 1131 **The Devil and Daniel**

The devil made a proposition to Daniel. The devil proposed paying Daniel for services in the following way: On the first day, I will pay you $1000 early in the morning. At the end of the day, you must pay me a commision of $100. At the end of the day, we will calculate your next day's salary and my commision. I will double what you have at the end of the day, but you must double the amount that you pay me. Will you work for me for a month?

PART A

Note: "...what you have..." is ambiguous. Ask for clarification if needed.

1. After reading the salary proposal, decide if you would work if you were Daniel. Write down your answer.

   ______________________________________________

2. Complete the following chart.

| Day | Salary for Daniel | | Commision for Devil | | Net Pay at the End of Day | |
|---|---|---|---|---|---|---|
| 1 | $1000 | | − $100 | | $ 900 | |
| 2 | $1800 | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |
| 7 | | | | | | |
| 8 | | | | | | |
| 9 | | | | | | |
| 10 | | | | | | |
| 11 | | | | | | |
| 12 | | | | | | |
| 13 | | | | | | |
| | | | | | | |
| ... | | | | | | |
| | | | | | | |
| 30 | | | | | | |
| | | | | | | |
| | | | | | | |

3. On the basis of the table you completed, would you stand by your decision?

_________________ Explain why or why not. _________________

_______________________________________________________________

_______________________________________________________________

4. If you would not work for a month, for how many days would you work?

_____________________

4b. How much money has Daniel collected on his Last day? (Add all Net Pay) (END OF DAY)

4c. If Daniel kept working, at the end of what day would the Devil be ahead? HINT: TOTAL SUM OF ALL OF DANIEL'S NET PAY WOULD BE NEGATIVE.

5. From reading the problem, what type of curve would you expect your salary data to generate?

_______________________________________________________________

6. From reading the problem, what type of curve would you expect the commission data to generate?

_______________________________________________________________

7. Is the salary scheme realistic?___________________________________

PART B.

**Discussion and Extension**

B 1. If you graphed your salary data, what type of graph would you expect to obtain?

_____________________ Why? ___________________________

_______________________________________________________________

B 2. In your own words, compare the graphs of the salary data and the commision data.

_______________________________________________________________

B3. Leaving everything else the same, how much salary would Daniel need on DAY 1 So he could work for 30 days? for 40 days?

B4. How many days would Daniel work if his salary were to be multiplied by 2.10 instead of 2 (no change for Devil)?

B5. To two decimals, what factor would Daniel's salary have to be multiplied for him to be able to work for a month?

B 6. Find an explicit formula for the sequence of net pay in the original problem, i.e. $a_n = \ldots$?

PRACTICE WITH SUMS FROM SEQUENCES

NAME: ____________

Period: ____

A. $\sum_{i=1}^{4}(3i+1)$

B. WRITE in $\sum$ NOTATION

$5+8+11+14+17$

C. FIND SUM FOR ARITHMETIC SEQUENCE WITH

$a_1=12$ and $a_{40}=129$

D. FIND SUM FOR GEOMETRIC SEQUENCE

$3+3\left(\frac{1}{4}\right)+3\left(\frac{1}{4}\right)^2+3\left(\frac{1}{4}\right)^3$

Sheet 1132 Solutions

PRACTICE WITH SUMS FROM SEQUENCES

NAME: KEY

Period: ___

A. $\sum_{i=1}^{4}(3i+1)=$

$a_1 = 3(1)+1 = 4$
$a_2 = 3(2)+1 = 7$
$a_3 = 10$
$a_4 = 13$

$= 4+7+10+13$
$= \boxed{34}$

B. WRITE in $\sum$ NOTATION

$5+8+11+14+17$

$\sum_{i=1}^{5} 5+(i-1)\cdot 3 = \boxed{\sum_{i=1}^{5}(2+3i)}$

C. FIND SUM FOR ARITHMETIC SEQUENCE WITH $a_1 = 12$ and $a_{40} = 129$

$S_{40} = \left(\frac{12+129}{2}\right)40 = 141\cdot 20 = \boxed{2820}$

D. FIND SUM FOR GEOMETRIC SEQUENCE

$3+3\left(\frac{1}{4}\right)+3\left(\frac{1}{4}\right)^2+3\left(\frac{1}{4}\right)^3$

$S_4 = 3\left(\frac{1-\left(\frac{1}{4}\right)^4}{1-\frac{1}{4}}\right) = 3\left(\frac{1-\frac{1}{256}}{3/4}\right) = 3\left(\frac{255/256}{3/4}\right) = 3\left(\frac{255}{256}\right)\left(\frac{4}{3}\right)$

$= \boxed{\frac{255}{64}} = 3.984375$

Name ______________________________ Per/Sec. ________________

State the first 5 terms of the sequence whose $n$th term is given by $a_n$.

1. $a_n = -\dfrac{3}{n}$

State the next 2 terms of the sequence and give a formula for the $n$th term.

2. 63, 54, 45, 36, 27

Find the sum.

3. $\sum_{n=1}^{4} n^4$

Express using sigma notation.

4. $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \frac{1}{81} - \frac{1}{243}$

5. State the 8th term of the geometric sequence $-9,\ \frac{9}{2},\ -\frac{9}{4}, \ldots$.

6. The first 3 terms of an arithmetic progression are $-15$, $-17$, and $-19$. Find the 22nd term and the sum of the first 22 terms.

7. How many terms of the arithmetic series $(-15) + (-8) + (-1) + \cdots$ must be added to give a sum of 801?

8. Find the sum of the first 6 terms of the geometric series $80 + (-20) + 5 + \cdots$.

9. $\frac{1}{2} + \left(-\frac{1}{4}\right) + \frac{1}{8} + \cdots$

10. A log pile has 61 logs in the bottom layer, 58 in the second layer, 55 in the third layer, and so on. If there are 639 logs in the pile, how many layers are there?

11. After the first swing, the path of a pendulum bob is 0.9 as long as long as the previous swing. If the first swing is 50 cm long, how far does the bob travel on the sixth swing? What total distance does the bob travel in those six swings?

12. **Formulas**
    - Arithmetic sequences and series: $a_n = a_1 + (n-1)d$ and $S_n = n\left(\dfrac{a_1 + a_n}{2}\right)$.
    - Geometric sequences and series: $a_n = a_1(r)^{n-1}$ and $S_n = a_1\left(\dfrac{1-r^n}{1-r}\right)$, and for $n \to \infty$ with $|r| < 1$, $S = a_1\left(\dfrac{1}{1-r}\right)$.

Sheet 1141: Sequences and Series Review

**Answer List**

1. $-3,\ -\frac{3}{2},\ -1,\ -\frac{3}{4},\ -\frac{3}{5}$
2. $18,\ 9;\ a_n = 72 - 9n$
3. $354$
4. $\sum_{1}^{6} \left(-\frac{1}{3}\right)^{k-1}$
5. $\frac{9}{128}$
6. $-57,\ -792$
7. $18$
8. $63\frac{63}{64}$
9. $\frac{1}{3}$
10. $18$
11. $\approx 29.5\,\text{cm};\ \approx 234.3\,\text{cm}$

Name:______________________________

# Sheet 1142: Power Series and Approximating a Function

Goals: Review Geometric Series, Study Power Series, Explore Approximating a Function by a Polynomial.

**Sequence**
**Series**
Story: Carl Friedrich Gauss (1777-1855)

1. Sum of first 100 whole numbers =

2. *Sum of whole numbers 41 to 70 =

**Geometric Series**

3. Bouncing Ball. $a = 1$, $r_{\text{ball}} \approx$ ______

| # × at peak | Height at peak | Sum of Heights |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| ⋮ | | |
| | | |

**Infinite Geometric Series**, $a = 1$

| $r$ | | $S$ |
|---|---|---|
| -1 | | |
| | | |
| 0 | | |
| | | |
| 1/2 | | |
| | | |
| 1 | | |

**Sum of Infinite Geometric Series**

Convergent Divergent

4. Find $S(r = 0.75) =$ _______

5. *Find $S(r_{\text{ball}}) =$ _______

**Power Series**, $r \to x$

6. $f(x) =$ ______ $=$ ________________

**Graph** $f(x)$ with TI-83 Plus calculator

7. Reset Defaults: [2nd][MEM (+)] 7 2 2

| 8. [WINDOW] | $f(x)$ | $\sin x$ (below) |
|---|---|---|
| Xmin= | -1 | ***-3*** |
| Xmax= | 1 | ***3*** |
| Xscl= | 0.25 | ***1*** |
| Ymin= | -1 | -1 |
| Ymax= | 4 | ***1*** |
| Yscl= | 1 | 1 |
| Xres= | 1 | 1 |

9. [Y=] $Y_1 = 1/(1-X)$
   $Y_2 = 1+X+X^2$ [GRAPH]
10. Keep adding $X^3$, $X^4$ etc. to $Y_2$= and compare the approximations to $f(x)$.
    Show the instructor. _________

**Graph** $\sin x$

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \ldots$$

11. [WINDOW] – See above table (part 8)
12. [Y=] $Y_1 = \sin(X)$
    $Y_2 = X - X^3/6$ [GRAPH]
13. Keep adding $X^5/120$, $-X^7/5040$

    $$\text{Also}: \cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \ldots$$

    $$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \ldots$$

One can use these to find Euler's formula:
$e^{ix} = \cos x + i \sin x$ (L. Euler 1707-1783).

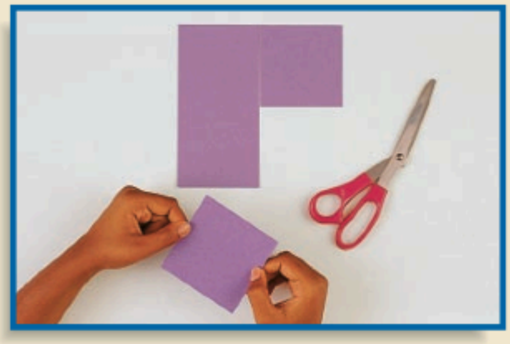
Name: ____________________________

## Sheet 1143: Cutting Paper to Infinity

You can illustrate an infinite geometric series by cutting a piece of paper into smaller and smaller pieces. Start with a square piece of paper. Define the area of the paper to be 1 square unit.

❶ Fold the paper in half and cut along the fold. Place one half on a desktop and hold the remaining half.

❷ Fold the piece of paper you are holding in half and cut along the fold. Place one half on the desktop and hold the remaining half.



❸ Repeat **Steps 1 and 2** until you find it too difficult to fold and cut the piece of paper you are holding.

❹ The first piece of paper has an area of ½ square unit. Write the areas of the next three pieces of paper:

Explain why these areas form a *geometric* sequence.

❺ Complete the table by recording the number of pieces of paper on the desktop and the combined area of the pieces at each step. No decimals!

| Number of pieces on desktop | 1 | 2 | 3 | 4 | 5 | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Combined area of pieces | ½ | ½ + ¼ = | | | | | | | |

❻ What number does the combined area of the pieces of paper appear to be approaching? ______

❼ What is the formula for the combined area after $n$ cuts? This is a sum of terms, $S_n$.

❽ Using the formula, what is the combined area for an infinite number of cuts, $n \to \infty$? Why?

❾ Using any method, find the sum for this infinite geometric series: $1/3 + 1/9 + 1/27 + \ldots$

Answer to 7: $\frac{1}{2}\left(\frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}}\right)$

## Sheet 1144: Arithmetic and Geometric Sequences & Series

Name: ____________________

1. Consider the following sequence: 12, 16, 20, 24, 28, 32….
(a) What is an explicit formula for the $n$th term, $a_n$?

(b) What is the value of the 2000th term, $a_{2000}$?

(c) What is the value of the sum of the first 2000 terms, $S_{2000}$?

2. Consider the following sequence: 4, 8, 16, 32,….
(a) What is an explicit formula for the $n$th term, $a_n$?

(b) What is the value of the 8th term, $a_8$?

(c) What is the value of the sum of the first 8 terms, $S_8$? Use the formula.

3. Consider the following sequence: 1/2, 1/4, 1/8, 1/16, ...
(a) What is the value of the 10th term, $a10$?

(b) What is the value of the sum of the first 10 terms, $S10$?

(c) What is the sum of the infinite series, $S$?

4. Find the sum $S$ for the infinite series 1/3 + 1/9 + 1/27, ...

5. Find the sum S for the infinite series 3/8 + 9/32 + 27/128, ...

| **Sheet 1151: Arithmetic, Geometric, & Recursive Sequences & Series** | Name: |
|---|---|

**Arithmetic.** Explicit: $a_n = a_1 + (n-1)d$ and $S_n = n\left(\dfrac{a_1 + a_n}{2}\right)$. Recursive: $a_1$ stated and $a_n = a_{n-1} + d$

**Geometric.** Explicit: $a_n = a_1(r)^{n-1}$, $S_n = a_1\left(\dfrac{1-r^n}{1-r}\right)$ and, for $n \to \infty$ with $|r|<1$, $S = a_1\left(\dfrac{1}{1-r}\right)$.

Recursive: $a_1$ stated, $a_n = a_{n-1} \cdot r$.

1. Write the next term in each sequence.

   (a) –3, –5, –7, –9, _______

   (b) 1, 4, 9, 16, 25, 36, _______

2. Circle each of the following sequences that is arithmetic.

   3, 6, 8, 12, 15, … 6, –2, –6, –10, …

   1, –1, 1, –1, 1, … $a_n = 3 \cdot (2)^{n-1}$

   $a_n = 14 - 6n$

3. Circle each of the following sequences that is geometric.

   3, 6, 12, 24, … 2, –2, –6, –10, …

   $a_n = 7n$ 1, –1, 1, –1, 1, …

   $a_n = 5 \cdot (4)^n$

4. Write the first five terms of the sequences with the following rules, starting with $n = 1$.

   (a) $a_n = 2n^2 - 3$

   (b) $\begin{cases} a_1 & = 3 \\ a_n & = a_{n-1} + 2 \end{cases}$

   (c) $\begin{cases} a_1 & = 1 \\ a_n & = 2a_{n-1} - n \end{cases}$

5. Find $\displaystyle\sum_{i=1}^{5} 2i$

6. Consider the following sequence: 14, 17, 20, 23, 26, 29….

   (a) What is an explicit formula for the $n$th term, $a_n$?

   (b) What is the value of the 2275th term, $a_{2275}$?

   (c) What is the value of the sum of the first 2275 terms, $S_{2275}$?

   (d) Write a recursive formula for the $n$th term, $a_n$. Recursive formulas have two equations.

7. Consider the following sequence: 2, 6, 18, 54,….

   (a) What is an explicit formula for the $n$th term, $a_n$?

   (b) What is the value of the 10th term, $a_{10}$? Show how you found the answer.

   (c) What is the value of the sum of the first 10 terms, $S_{10}$? For full credit, use a formula. You may check your results using a different method.

   (d) Write a recursive formula for the $n$th term, $a_n$. Recursive formulas have two equations.

ARITH, GEOM, RECURSIVE SEQ & SERIES

Sheet 1151 Name: $\boxed{\text{KEY}}$

**Arithmetic.** Explicit: $a_n = a_1 + (n-1)d$ and $S_n = n\left(\frac{a_1 + a_n}{2}\right)$. Recursive: $a_1$ stated and $a_n = a_{n-1} + d$

**Geometric.** Explicit: $a_n = a_1(r)^{n-1}$, $S_n = a_1\left(\frac{1-r^n}{1-r}\right)$ and, for $n \to \infty$ with $|r| < 1$, $S = a_1\left(\frac{1}{1-r}\right)$.

Recursive: $a_1$ stated, $a_n = a_{n-1} \cdot r$.

1. Write the next term in each sequence.

(a) $-3, -5, -7, -9,$ $\boxed{-11}$

$-2 \quad -2 \quad -2$

(b) $1, 4, 9, 16, 25, 36,$ $\boxed{49}$

2. Circle each of the following sequences that is arithmetic.

$3, 6, 8, 12, 15, \ldots$ $\qquad$ $6, -2, -6, -10, \ldots$

$1, -1, 1, -1, 1, \ldots$ $\qquad$ $a_n = 3 \cdot (2)^{n-1}$

$a_n = 14 - 6n$

3. Circle each of the following sequences that is geometric.

$3, 6, 12, 24, \ldots$ $\qquad$ $2, -2, -6, -10, \ldots$

$a_n = 7n$ $\qquad$ $1, -1, 1, -1, 1, \ldots$

$a_n = 5 \cdot (4)^n$

4. Write the first five terms of the sequences with the following rules, starting with $n = 1$.

(a) $a_n = 2n^2 - 3$

$a_1 = -1$
$a_2 = 5$
$a_3 = 15$
$a_4 = 29$
$a_5 = 47$

(b) $\begin{cases} a_1 = 3 \\ a_n = a_{n-1} + 2 \end{cases}$

$a_1 = 3$
$a_2 = 5$
$a_3 = 7$
$a_4 = 9$
$a_5 = 11$

(c) $\begin{cases} a_1 = 1 \\ a_n = 2a_{n-1} - n \end{cases}$

$a_1 = 1$
$a_2 = 0$
$a_3 = -3$
$a_4 = -10$
$a_5 = -25$

5. The answer to the new Question 5:
Sum = 2 + 4 + 6 + 8 + 10 = 30.

6 ~~5~~ Consider the following sequence: 14, 17, 20, 23, 26, 29....

11 $\quad$ +3

(a) What is an explicit formula for the $n$th term, $a_n$?

$\boxed{a_n = 14 + (n-1) \cdot 3}$

$a_n = 11 + 3n$

(b) What is the value of the 2275th term, $a_{2275}$?

$\boxed{6836}$

(c) What is the value of the sum of the first 2275 terms, $S_{2275}$?

$S_{2275} = \frac{1}{2}(14 + 6836) \cdot 2275 =$ $\boxed{7,791,875}$

(d) Write a recursive formula for the $n$th term, $a_n$. Recursive formulas have two equations.

$a_1 = 14$
$a_n = a_{n-1} + 3$

7 ~~6~~ Consider the following sequence: 2, 6, 18, 54,....

(a) What is an explicit formula for the $n$th term, $a_n$?

$\boxed{a_n = 2(3)^{n-1}}$

$a_n = (2/3)\, 3^n$

(b) What is the value of the 10th term, $a_{10}$? Show how you found the answer.

$a_{10} = 2(3)^9 =$ $\boxed{39366}$ $= 2(3)^9$

(c) What is the value of the sum of the first 10 terms, $S_{10}$? For full credit, use a formula. You may check your results using a different method. (Answer as an improper fraction.)

$S_{10} = 2\left(\frac{1 - 3^{10}}{1 - 3}\right) =$ $\boxed{59{,}048}$

(d) Write a recursive formula for the $n$th term, $a_n$. Recursive formulas have two equations.

$a_1 = 2$
$a_n = a_{n-1} \cdot 3$

**Sheet 1152** Name: ______________________ Partners: ______________ ______________ Score: ___

## Bouncing Ball Investigation

Report to be handed in. Copies: http://teach.kralsite.com/ *Dr. Fred Kral, v. 2.11*

**Materials**: 152-cm measuring tape (or longer), bouncing ball, tape, calculator.

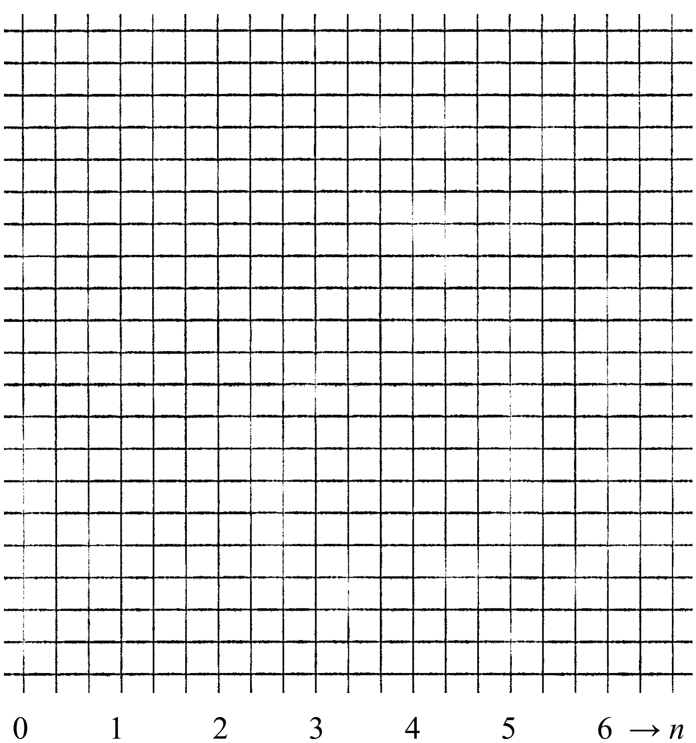
**Goal**: Find a model describing the rebound heights of a bouncing ball as a geometric sequence.

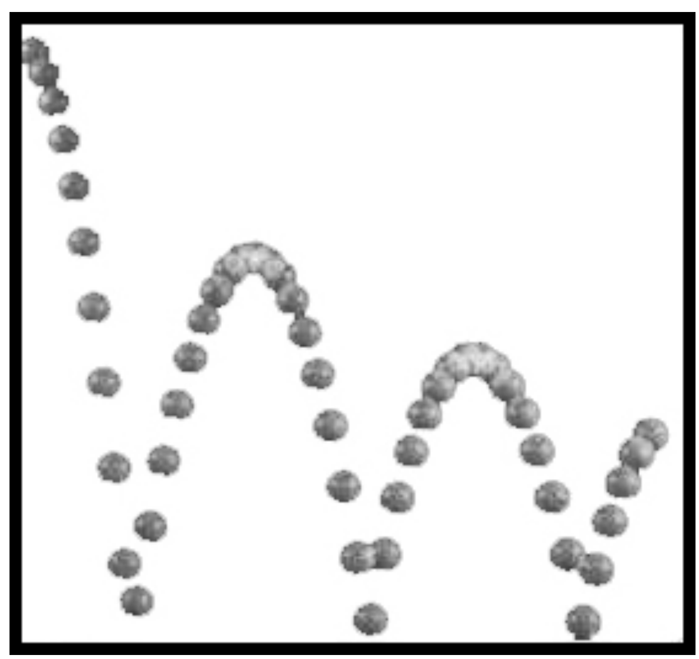
**Gather Data**: First write down the **ball number:** ____________. Then gather your materials and locate a big space of hard flat ground near a wall. Tape the measuring tape to the wall with 0 cm at the bottom. (Note numbers are above the line on the measuring tape). Drop the ball from 150 cm, and record the *"rebound height"* – the maximum height that the ball reaches after each bounce. Do five (5) trials for each number of bounces up to six (6) rebounds. That is, 5 trials for Rebound 1, 5 trials for Rebound 2, etc.

| **Re-bound Number** | **Rebound Height (Max. Height after N bounces)** | | | | | **Average Rebound Height** | **Rebound Ratio, *r*** | Predicted Rebound Height |
|---|---|---|---|---|---|---|---|---|
| | Trial 1 | Trial 2 | Trial 3 | Trial 4 | Trial 5 | | | |
| 0 | — | — | — | — | — | 150 cm | — | 150 cm |
| 1 | | | | | | | | |
| 2 | | | | | | | | |
| 3 | | | | | | | | |
| 4 | | | | | | | | |
| 5 | | | | | | | | |
| 6 | | | | | | | | |

**Plot Data**: Calculate and plot Average Rebound Height, $H_n$ ($y$-axis) versus Rebound Number, $n$, ($x$-axis).

**Average Rebound Height versus Rebound Number**



Name: ________________ Partners: __________ _________ Period: __

**Sheet 1152: Bouncing Ball Investigation, continued**

**Questions and data analysis:**

If each average rebound height is $H_n$, we can set the starting height to be $H_0 = 150$ cm, with $n = 0$.

1. If you have not already done so, compute an estimated Rebound Ratio for this ball, $r$, for each successive pair of Average Rebound Heights: $r = \dfrac{\text{Average Rebound Height}}{\text{Previous Average Rebound Height}} = \dfrac{H_n}{H_{n-1}}$. Put $r$ values in the table.

2. Do you think the rebound heights may make up a geometric sequence? __________. Ideally, the ratios in your table will be the same for each pair of rebounds. In your experiment this will probably not be exactly true. Decide on a single value for $r$ that best represents the rebound ratio for your ball.
Best value for $r$: ___________. How did you calculate it? ____________________________________.

3. Write a recursive formula that uses this common ratio, $r$. Note that we are starting at $n = 0$.
First use an equation with the words **THIS** term and **PREVIOUS** term:___________________________

Then write a symbolic recursive formula: $H_o =$
$H_n =$

4. Fill out the last column in your table using the best value for $r$ and the formula. The first entry is your starting height. The next entry is a calculated Predicted Rebound Height, which is 150 cm × $r$, etc.

5. Calculate differences between the measurement and the mathematical model: Average − Prediction.
Is the model "good?" ______ . How large are the differences? ____________.
Would another "best" value of $r$ perhaps fit the data better? ______ If, so what $r$ would work better? ______.

6. We will now work towards writing the explicit formula.
First, using only $H_0$ and $r$, write the equation for the first rebound, $H_1 =$
Then, write the second term, using only $H_0$ and $r$ (no numbers), $H_2 =$

What would be the equation of the **fifth** term, using only $H_0$ and $r$, $H_5 =$

Finally, write the equation of the ***n*th** term, using only $H_0$ and $r$, $H_n =$
This is the explicit formula. Check that it gives the predicted values.

**Exponential Regression—using the calculator to find the best fit:**

7. Enter the $H_n$ versus $n$ data in the TI-83 calculator.
Use [STAT] 1: Edit…. Put $n$ into List 1 ($L_1$). Put $H_n$ into List 2 ($L_2$). **Start with $n = 0$ and $H_0 = 150$.**

8. Plot the data on the calculator.
Use [2nd] [STAT PLOT] 1: Plot 1. Turn ON and use the First Type of plot (scatter). XList: $L_1$ YList $L_2$
Before you plot, do [ZOOM] 9: ZoomStat.
[GRAPH] Change the [WINDOW] to Xmin=-1, Xmax=7, Xscl=1, Ymin=0, Ymax=170, Yscl=10.
[GRAPH] **Show the teacher.**

9. Find the best fit by using exponential regression (a statistical method), $y = ab^x$
Use [STAT] CALC (right arrow) 0: ExpReg.
What is your best fit value of $a$? $a =$ _________. What is your best fit value of $b$? $b =$ __________.
What is the meaning of $a$ in this experiment? __________. What is the meaning of $b$? _____________.

10. Put the function $y = ab^x$ into the calculator. Use the [Y=] button. Push [GRAPH]. **Show the teacher.**
How are exponential functions and geometric sequences related? ____________________________.

Example of **Investigation 2.3.1** - Bouncing Ball (solid black superball)

| | Maximum Height Data --Trial Number | | | | | | | | r= 0.84 |
|---|---|---|---|---|---|---|---|---|---|
| Bounces | 1 | 2 | 3 | 4 | 5 | **Average** | Difference | **Ratio** | 200*(r)^n |
| 0 | | | | | | **200.0** | | | 200.0 |
| 1 | 161 | 163 | 162 | 161 | 160 | **161.4** | -38.6 | **0.81** | 168.0 |
| 2 | 133 | 135 | 136 | 136 | 133 | **134.6** | -26.8 | **0.83** | 141.1 |
| 3 | 115 | 117 | 118 | 119 | 116 | **117.0** | -17.6 | **0.87** | 118.5 |
| 4 | 100 | 98 | 97 | 98 | 98 | **98.2** | -18.8 | **0.84** | 99.6 |
| 5 | 87 | 87 | 85 | 87 | 88 | **86.8** | -11.4 | **0.88** | 83.6 |
| 6 | 80 | 77 | 78 | 79 | 79 | **78.6** | -8.2 | **0.91** | 70.3 |

| Raw data | | | 200 minus height | | |
|---|---|---|---|---|---|
| 1 | 39 | 37 | 38 | 39 | 40 |
| 2 | 67 | 65 | 64 | 64 | 67 |
| 3 | 85 | 83 | 82 | 81 | 84 |
| 4 | 100 | 102 | 103 | 102 | 102 |
| 5 | 113 | 113 | 115 | 113 | 112 |
| 6 | 120 | 123 | 122 | 121 | 121 |