NAME: ______________________________ BLOCK/PERIOD: ______

RAT PACKET

STARRING:

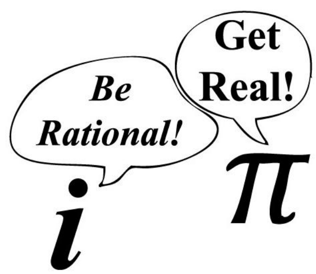
RATIONAL FUNCTIONS

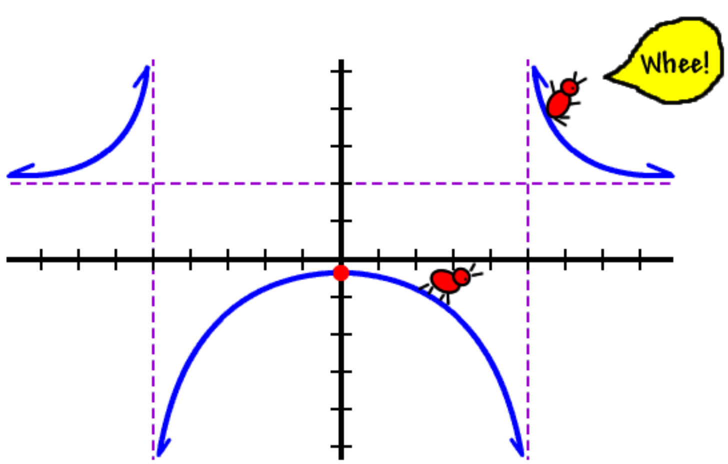
AND

THEIR GRAPHS

ALSO FEATURING:

POLYNOMIAL FUNCTIONS AND RATIONAL EXPRESSIONS





v.3

Sheet 931: Polynomial Function Questions

FMC 2nd Ch 9.2-9.3, pp. 388, 394, 395.

9.2:

7. Estimate the zeros of $f(x) = x^4 - 3x^2 - x + 2$.

8. Estimate the minimum value of $g(x) = x^4 - 3x^3 - 8$.

Describe in words the long-run behavior as $x \to \infty$ of the functions in Exercises 9–12. What power function does each resemble?

9. $y = 16x^3 - 4023x^2 - 2$

10. $y = 4x^4 - 2x^2 + 3$

11. $y = 3x^3 + 2x^2/x^{-7} - 7x^5 + 2$

12. $y = 5x^2/x^{3/2} + 2$

15. Find the equation of the line through the y-intercept of $y = x^4 - 3x^5 - 1 + x^2$ and the x-intercept of $y = 2x - 4$.

9.3:

4. Factor $f(x) = 8x^3 - 4x^2 - 60x$ completely, and determine the zeros of f.

In Exercises 5–8, find the zeros of the functions.

5. $y = 7(x+3)(x-2)(x+7)$

6. $y = a(x+2)(x-b)$, where a, b are nonzero constants

7. $y = x^3 + 7x^2 + 12x$

8. $y = (x^2 + 2x - 7)(x^3 + 4x^2 - 21x)$

11. Without using a calculator, decide which of the equations A–E best describes the polynomial in Figure 9.26.

A $y = (x+2)(x+1)(x-2)(x-3)$
B $y = x(x+2)(x+1)(x-2)(x-3)$
C $y = -\frac{1}{2}(x+2)(x+1)(x-2)(x-3)$
D $y = \frac{1}{2}(x+2)(x+1)(x-2)(x-3)$
E $y = -(x+2)(x+1)(x-2)(x-3)$

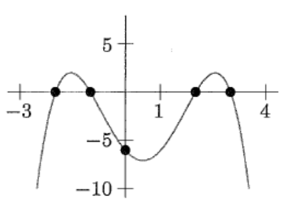
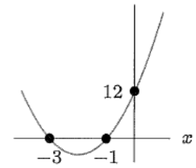


Figure 9.26

Give a possible formula for the polynomials in Questions 12–25.

12.



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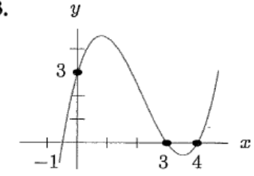

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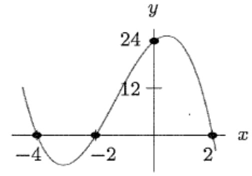

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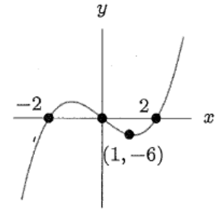

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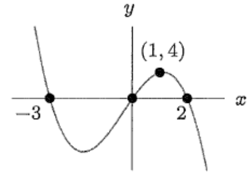

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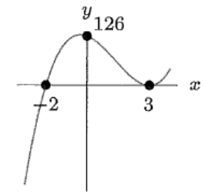

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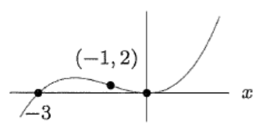

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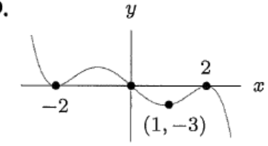

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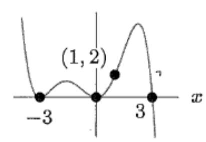

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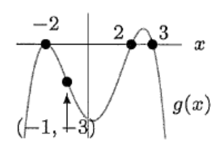

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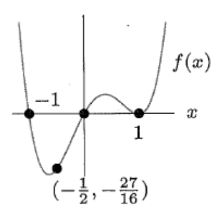

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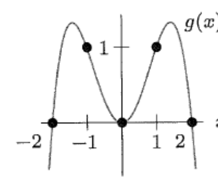

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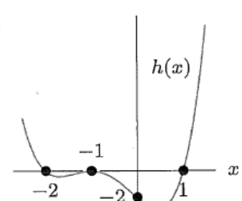

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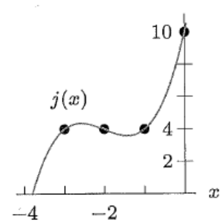


For Problems 34–39, find the real zeros (if any) of the polynomials.

34. $y = x^2 + 5x + 6$

35. $y = x^4 + 6x^2 + 9$

36. $y = 4x^2 - 1$

37. $y = 4x^2 + 1$

38. $y = 2x^2 - 3x - 3$

39. $y = 3x^5 + 7x + 1$

Ch. 9.2: 7. 0.718, 1.702. 9. x^3. 11. x^9. 15. $y = 0.5x - 1$.
Ch. 9.3: 5. −3, 2, −7. 7. 0, −4, −3. 11. C.

12 $y = 4(x+1)(x+3)$

13 $y = \frac{1}{2}(x+\frac{1}{2})(x-3)(x-4)$

14 $y = -\frac{3}{2}(x+4)(x+2)(x-2)$

15 $y = 2x(x+2)(x-2)$

16 $y = -x(x+3)(x-2)$.

17 $y = 7(x+2)(x-3)^2$.

18 $y = x^2(x+3)$

19 $y = -\frac{1}{3}(x+2)^2(x)(x-2)^2$.

20 $f(x) = -\frac{1}{16}(x+3)^2(x-3)x^2$

21 $g(x) = -\frac{1}{4}(x+2)^2(x-2)(x-3)$

22 $f(x) = 3x(x+1)(x-1)^2$

23 $g(x) = -\frac{1}{3}(x+2)(x-2)x^2$

24. $h(x) = (x+2)(x+1)^2(x-1)$ 25. $j(x) = (x+3)(x+2)(x+1) + 4$

35. no. 37. no. 39. −0.143.

NAME: ____________ Per. ____

Sheet #941: Hyperbolic Function Graph

1. CONSIDER: $y = \dfrac{2x+3}{x-2}$

a, GRAPH ON TI.

$\boxed{\text{2nd}}\ \boxed{\text{MEM } +}$ 7 2 2

$\boxed{\text{MODE}}$ Dot

$\boxed{\text{ZOOM}}$ 6

b, SKETCH USING A TABLE, $\boxed{\text{2nd}}\ \boxed{\text{TABLE Graph}}$
AND MANY POINTS.

c, IDENTIFY VERTICAL ASYMPTOTE.
WRITE EQUATION

d, FIND INTERCEPTS
Zero (x-intercept): y-intercept:

e, IDENTIFY HORIZONTAL Asymptote
you may need to enlarge $\boxed{\text{WINDOW}}$.
WRITE EQUATION:

f, GRAPH $y = \dfrac{3x+2}{x-2}$ on TI.

g, MAKE A CONJECTURE ABOUT HORIZ. Asymptote.

NAME: $\boxed{\text{KEY}}$ Per. ____

Sheet #941 : Hyperbolic Function Graph

1. CONSIDER: $y = \dfrac{2x+3}{x-2}$

a, GRAPH ON TI.

$\boxed{\text{2nd}}\boxed{\text{MEM} +}$ 7 2 2

$\boxed{\text{MODE}}$ Dot

$\boxed{\text{ZOOM}}$ 6

b, SKETCH USING A TABLE, $\boxed{\text{2nd}}\boxed{\text{TABLE Graph}}$
AND MANY POINTS.

c, IDENTIFY VERTICAL ASYMPTOTE
WRITE EQUATION $x = 2$

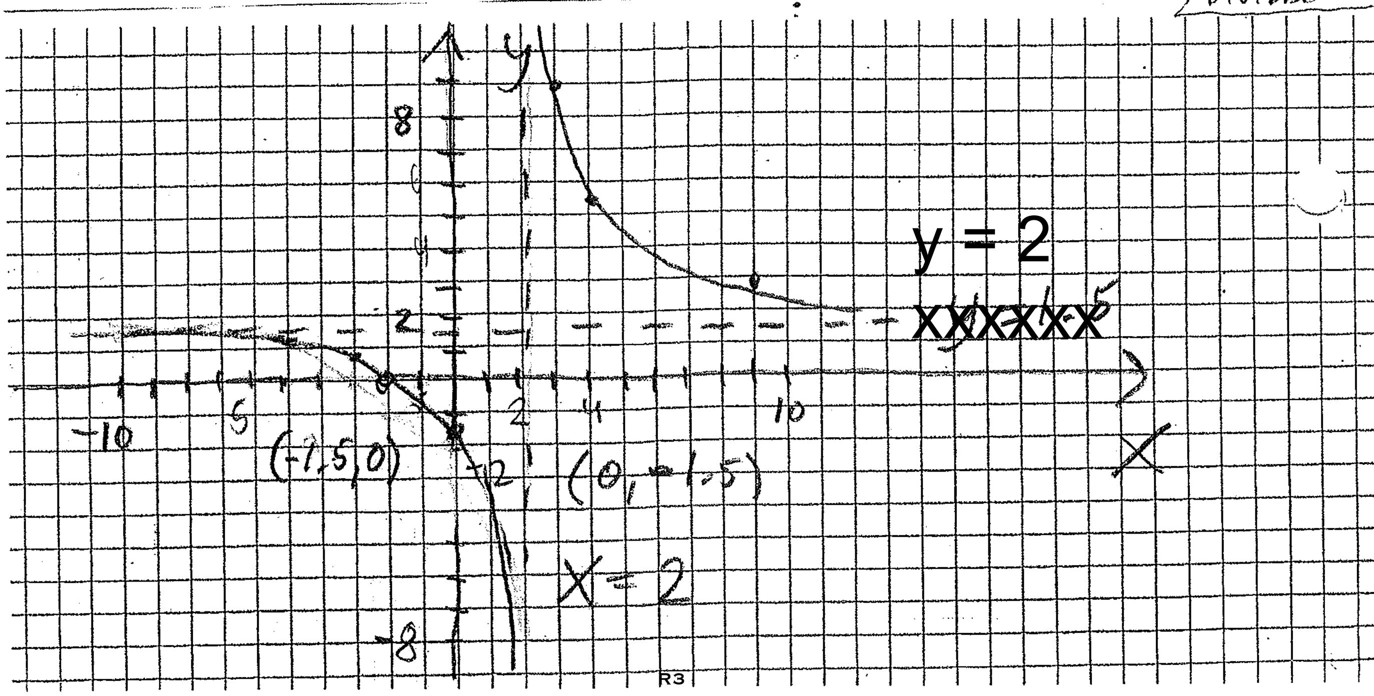
d, FIND INTERCEPTS
Zero (x-intercept): $\left(-\frac{3}{2}, 0\right)$ y-intercept $\left(0, -\frac{3}{2}\right)$

e, IDENTIFY HORIZONTAL Asymptote
you may need to enlarge $\boxed{\text{WINDOW}}$.
WRITE EQUATION: $y = 2$

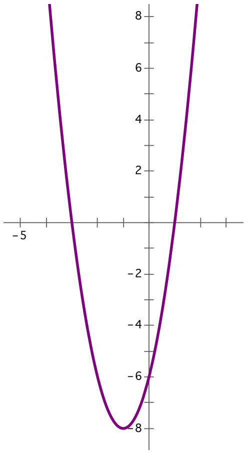
f, GRAPH $y = \dfrac{3x+2}{x-2}$ ON TI.

g, MAKE A CONJECTURE ABOUT HORIZ. Asymptote
$y = 3/2$ IT IS $\frac{3x}{x}$ VS. $\frac{2x}{x}$ So COEFF. OF LEADING TERMS DIVIDED

METHODS
(c) *DENOMINATOR: Set $x-2=0$
(d) *NUMERATOR: Set $y=0$
*FULL FUNCTION: Set $x=0$
(g) RATIO OF LEADING TERMS.



Sheet 951: Find Possible Formulas of Polynomial and Rational Functions from their Graphs   Name: ________________



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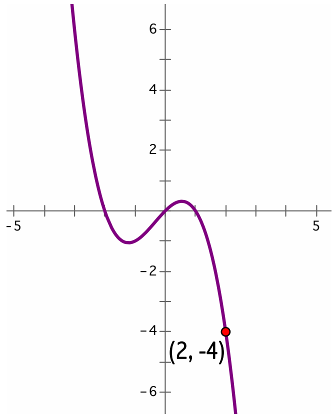

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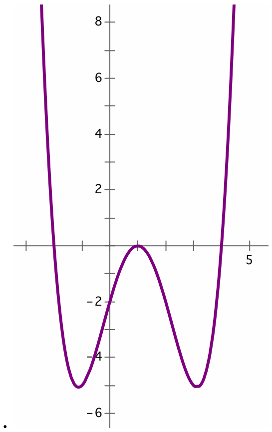

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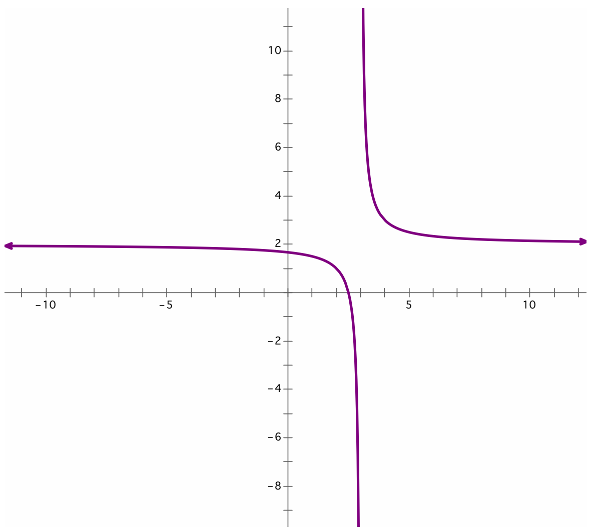

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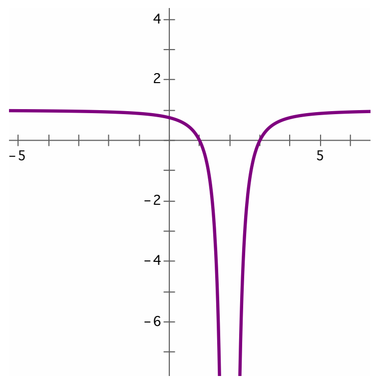

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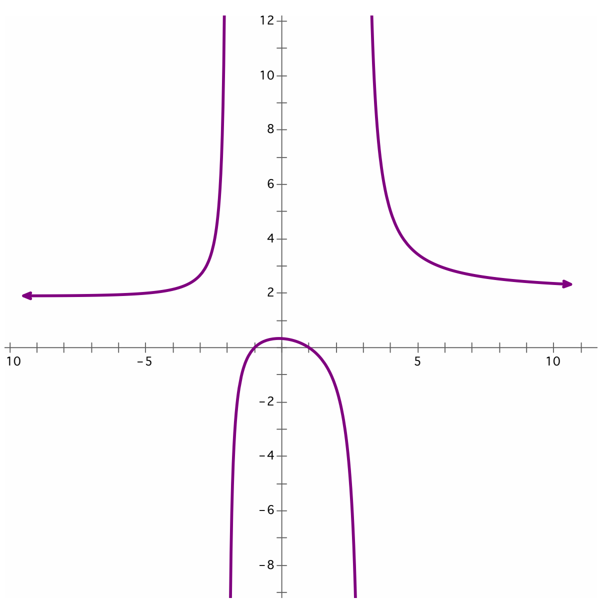

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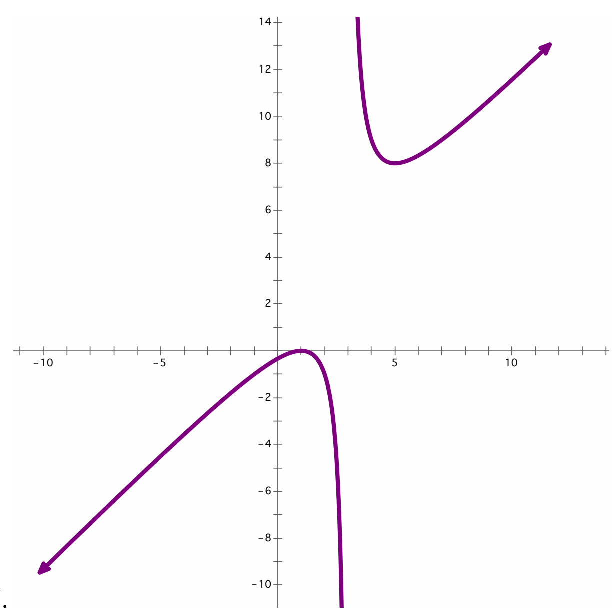


7.

$f(x) = 2\cdot(x-1)\cdot(x+3)$ $g(x) = -0.5\cdot x\cdot(x+2)\cdot(x-1)$ $h(x) = 0.25\cdot(x-1)^2\cdot(x-4)\cdot(x+2)$

$q(x) = \dfrac{1}{x-3} + 2$ $r(x) = \dfrac{-1}{(x-2)^2} + 1$ $s(x) = \dfrac{2\cdot(x+1)\cdot(x-1)}{(x+2)\cdot(x-3)}$ $t(x) = \dfrac{(x-1)^2}{x-3}$

Sheet 955: POLYNOMIAL AND RATIONAL FUNCTIONS.

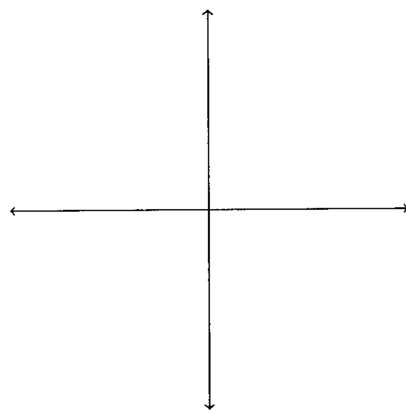
Name ______________________________

Make sure to know how to do multiple choice questions without choices given.

Graphing calculator not allowed

1. Find the simplest polynomial function with integer coefficients whose graph crosses the x-axis only at $(-5,0)$, $(5,0)$, $(-2,0)$, and $(2,0)$.

2. Write asymptotes and intercepts. Sketch. $g(x) = \dfrac{x+2}{3-x}$



Write asymptotes and intercepts.

3. $g(x) = \dfrac{-1}{x-4}$

4. $h(x) = \dfrac{5x-2}{x}$

5. $m(x) = \dfrac{4}{(x-5)^2}$

6. $g(x) = \dfrac{x+2}{(x-2)(x+7)}$

7. Write slant asymptote. $q(x) = \dfrac{5x^2+3x-2}{x-2}$

Answer List

1. $g(x) = x^4 - 29x^2 + 100$
2. $x = 3$, $y = -1$
3. $x = 4$, $y = 0$
4. $x = 0$, $y = 5$
5. $x = 5$, $y = 0$
6. $x = 2$, $x = -7$, $y = 0$
7. $y = 5x + 13$ OR JUST $y=5x$
8. 16 items; \$500
9. c
10. e

Graphing calculator required

8. The total daily profit P, in dollars on the production of x items is $P(x) = -2x^2 + 64x - 12$. How many items must be produced each day to maximize profit? What is the maximum profit?

9. Which of the following is an approximate zero of $y = 10x^3 - 9x + 4$?

a) -1.4597 b) -1.2201 c) -1.1211 d) -0.5280 e) 1.3265

10. Given $f(x) = x^2 - 1$ and $g(x) = \dfrac{1}{x}$, find the x-value of the point of intersection of the two functions.

a) 0.358 b) 0.754 c) 0.897 d) 1.134 e) 1.325

Sheet 991: Rational Expressions Practice
FMC 2nd Ch 9 Tools, p. 433–434

For Exercises 1–35, perform the operations. Express answers in reduced form.

1. $\frac{3}{5}+\frac{4}{7}$

2. $\frac{7}{10}-\frac{2}{15}$

3. $\frac{1}{2x}-\frac{2}{3}$

4. $\frac{6}{7y}+\frac{9}{y}$

5. $\frac{-2}{yz}+\frac{4}{z}$

6. $\frac{-2z}{y}+\frac{4}{y}$

7. $\frac{2}{x^2}-\frac{3}{x}$

8. $\frac{6}{y}+\frac{7}{y^3}$

9. $\frac{3/4}{7/20}$

10. $\frac{5/6}{15}$

11. $\frac{3/x}{x^2/6}$

12. $\frac{3/x}{6/x^2}$

13. $\frac{13}{x-1}+\frac{14}{2x-2}$

14. $\frac{14}{x-1}+\frac{13}{2x-2}$

15. $\frac{4z}{x^2y}-\frac{3w}{xy^4}$

16. $\frac{10}{y-2}+\frac{3}{2-y}$

17. $\frac{8y}{y-4}+\frac{32}{y-4}$

18. $\frac{8y}{y-4}+\frac{32}{4-y}$

19. $\dfrac{\frac{1}{x}-\frac{2}{x^2}}{\frac{2x-4}{x^5}}$

20. $\frac{9}{x^2+5x+6}+\frac{12}{x+3}$

21. $\frac{8}{3x^2-x-4}-\frac{9}{x+1}$

22. $\frac{5}{(x-2)^2(x+1)}-\frac{18}{(x-2)}$

23. $\frac{15}{(x-3)^2(x+5)}+\frac{7}{(x-3)(x+5)^2}$

24. $\frac{3}{x-4}-\frac{2}{x+4}$

25. $\frac{x^2}{x-1}-\frac{1}{1-x}$

26. $\frac{1}{2r+3}+\frac{3}{4r^2+6r}$

27. $u+a+\frac{u}{u+a}$

28. $\frac{1}{\sqrt{x}}-\frac{1}{(\sqrt{x})^3}$

29. $\frac{1}{e^{2x}}+\frac{1}{e^x}$

30. $\frac{a+b}{2}\cdot\frac{8x+2}{b^2-a^2}$

31. $\frac{0.07}{M}+\frac{3}{4}M^2$

32. $\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$

33. $\frac{8y}{y-4}-\frac{32}{y-4}$

34. $\frac{a}{a^2-9}+\frac{1}{a-3}$

35. $\frac{x^3}{x-4}\Big/\frac{x^2}{x^2-2x-8}$

In Exercises 36–49, simplify, if possible.

36. $\frac{1/(x+y)}{x+y}$

37. $\frac{(w+2)/2}{w+2}$

38. $\dfrac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$

39. $\frac{a^{-2}+b^{-2}}{a^2+b^2}$

1 $41/35$
3 $(3-4x)/6x$
5 $-2(1-2y)/yz$
7 $(2-3x)/x^2$
9 $15/7$
11 $18/x^3$
13 $20/(x-1)$
15 $(4y^3z-3wx)/(x^2y^4)$
17 $(8(y+4))/(y-4)$

19 $x^3/2$
21 $(-27x+44)/((x+1)(3x-4))$
23 $2(11x+27)/((x-3)^2(x+5)^2)$
25 $(x^2+1)/(x-1)$
27 $\left((u+a)^2+u\right)/(u+a)$
29 $(1+e^x)/e^{2x}$
31 $(0.28+3M^3)/(4M)$
33 8
35 $x(x+2)$
37 $1/2$
39 $1/(a^2b^2)$

40. $\dfrac{a^2-b^2}{a^2+b^2}$

41. $\dfrac{4-(x+h)^2-(4-x^2)}{h}$

42. $\dfrac{b^{-1}(b-b^{-1})}{b+1}$.

43. $\dfrac{1-a^{-2}}{1+a^{-1}}$.

44. $\dfrac{x^{-1}+x^{-2}}{1-x^{-2}}$.

45. $p-\dfrac{q}{\dfrac{p}{q}+\dfrac{q}{p}}$

46. $\dfrac{\dfrac{3}{xy}-\dfrac{5}{x^2y}}{\dfrac{6x^2-7x-5}{x^4y^2}}$

47. $\dfrac{\dfrac{1}{x}\left(3x^2\right)-(\ln x)(6x)}{\left(3x^2\right)^2}$

48. $\dfrac{2x(x^3+1)^2-x^2(2)(x^3+1)(3x^2)}{[(x^3+1)^2]^2}$

49. $\dfrac{\frac{1}{2}(2x-1)^{-1/2}(2)-(2x-1)^{1/2}(2x)}{(x^2)^2}$

In Exercises 50–55, split into a sum or difference of reduced fractions.

50. $\dfrac{26x+1}{2x^3}$

51. $\dfrac{\sqrt{x}+3}{3\sqrt{x}}$

52. $\dfrac{6l^2+3l-4}{3l^4}$

53. $\dfrac{7+p}{p^2+11}$

54. $\dfrac{\frac{1}{3}x-\frac{1}{2}}{2x}$

55. $\dfrac{t^{-1/2}+t^{1/2}}{t^2}$

In Exercises 56–61, rewrite in the form $1+(A/B)$.

56. $\dfrac{x-2}{x+5}$

57. $\dfrac{q-1}{q-4}$

58. $\dfrac{R+1}{R}$

59. $\dfrac{3+2u}{2u+1}$

60. $\dfrac{\cos x+\sin x}{\cos x}$

61. $\dfrac{1+e^x}{e^x}$

Are the statements in Exercises 62–67 true or false?

62. $\dfrac{a+c}{a}=1+c$

63. $\dfrac{rs-s}{s}=r-1$

64. $\dfrac{y}{y+z}=1+\dfrac{y}{z}$

65. $\dfrac{2u^2-w}{u^2-w}=2$

66. $\dfrac{x^2yz}{2x^2y}=\dfrac{z}{2}$

67. $x^{5/3}-3x^{2/3}=\dfrac{x^2-3x}{x^{1/3}}$

41 $-2x-h$
43 $1-(1/a)$
45 $p^3/(p^2+q^2)$
47 $(1-2\ln x)/\left(3x^3\right)$
49 $(-4x^2+2x+1)/(x^4\sqrt{2x-1})$
51 $1/3+1/\sqrt{x}$
53 $7/(p^2+11)+p/(p^2+11)$
55 $1/t^{5/2}+1/t^{3/2}$
57 $1+3/(q-4)$
59 $1+2/(2u+1)$
61 $1+1/e^x$

62. False
63. True
64. False
65. False
66. True
67. True, factor $x^{-1/3}$

LESSON **9.3**

NAME ______________________ DATE ______

Practice with Examples

For use with pages 547–553

9.3A

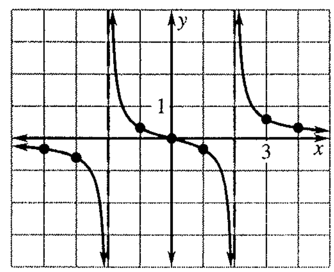
GOAL **Graph general rational functions**

EXAMPLE 1 *Graphing a Rational Function* ($m < n$)

Graph $y = \dfrac{x}{x^2 - 4}$.

Solution

Begin by finding the x-intercepts of the graph, which are the real zeros of the numerator. The numerator has 0 as its only zero, so the graph has one x-intercept at (0, 0). Then find the vertical asymptotes of the graph, which are the real zeros of the denominator. The denominator contains a difference of two squares that can be factored as $(x - 2)(x + 2)$, so the denominator has zeros 2 and -2, and the graph has vertical asymptotes $x = 2$ and $x = -2$. Finally, determine if the graph has a horizontal asymptote. Because the degree of the numerator (1) is less than the degree of the denominator (2), the line $y = 0$ is a horizontal asymptote. Construct a table of values consisting of x-values between and beyond the vertical asymptotes.



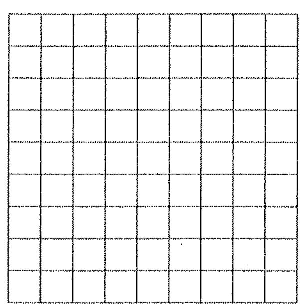
x	-4	-3	-1	0	1	3	4
y	$-\frac{1}{3}$	$-\frac{3}{5}$	$\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{3}{5}$	$\frac{1}{3}$

Plot the points and use the asymptotes to draw the graph.

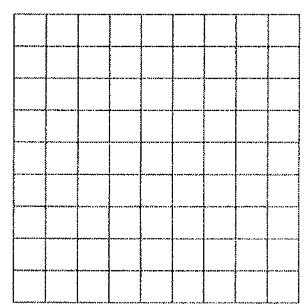
Exercises for Example 1

Graph the function.

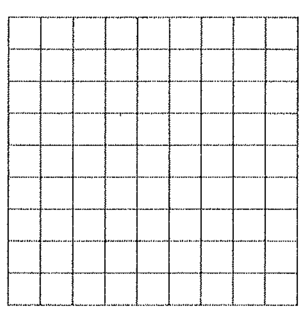
1. $y = \dfrac{2}{x + 1}$



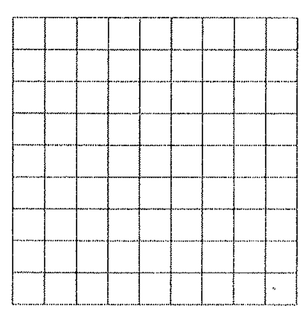
2. $y = \dfrac{x}{x^2 - 1}$



3. $y = \dfrac{-3}{x - 2}$



4. $y = \dfrac{2x}{x^2 + 4}$



Chapter 9

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LESSON **9.3** CONTINUED

9.3B

NAME ______________________________ DATE __________

Practice with Examples

For use with pages 547–553

EXAMPLE 2 *Graphing a Rational Function* ($m = n$)

Graph $y = \dfrac{x^2 + 2}{x^2 - x - 6}$.

SOLUTION

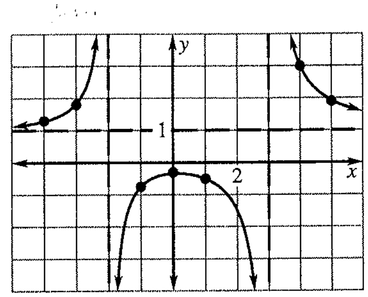
The numerator has no zeros, so there are no x-intercepts. The denominator can be factored as $(x - 3)(x + 2)$, so the denominator has zeros 3 and -2 and the graph has vertical asymptotes $x = 3$ and $x = -2$. Because the degree of the numerator (2) equals the degree of the denominator (2), the line

$y = \dfrac{a_m}{b_n} = \dfrac{1}{1} = 1$ is a horizontal asymptote.

Construct a table of values consisting of x-values between and beyond the vertical asymptotes.

x	-4	-3	-1	0	1	4	5
y	1.3	1.8	-0.75	-0.3	-0.5	3	1.9

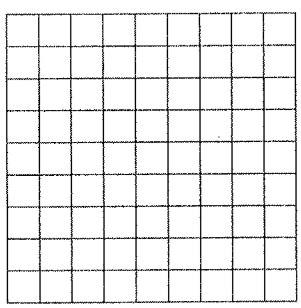
Plot the points and use the asymptotes to draw the graph.



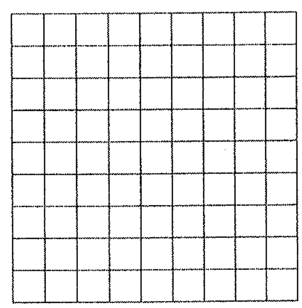
Exercises for Example 2

Graph the function.

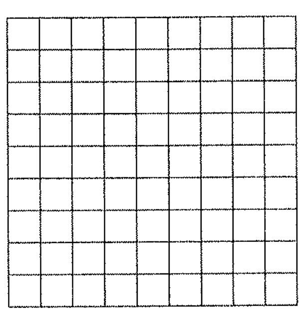
5. $y = \dfrac{x + 1}{x - 3}$



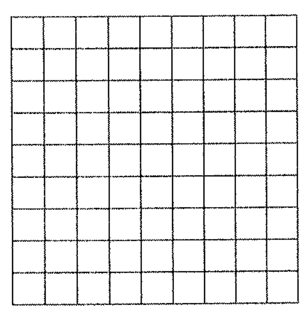
6. $y = \dfrac{2x + 3}{x + 2}$



7. $y = \dfrac{3x^2}{x^2 + 9}$



8. $y = \dfrac{2x^2}{x^2 - 1}$



Chapter 9

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LESSON **9.3** CONTINUED

9.3C

NAME ______________________ DATE __________

Practice with Examples

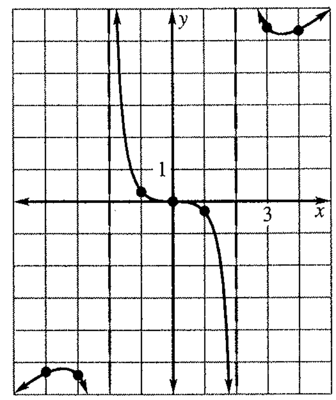
For use with pages 547–553

EXAMPLE 3 *Graphing a Rational Function* $(m > n)$

Graph $y = \dfrac{x^3}{x^2 - 4}$.

SOLUTION

The numerator has 0 as its only zero, so the x-intercept of the graph is $(0, 0)$. The denominator can be factored as $(x + 2)(x - 2)$, so the graph has vertical asymptotes $x = 2$ and $x = -2$. The degree of the numerator (3) is greater than the degree of the denominator (2), so there is no horizontal asymptote. But, the end behavior of the graph is the same as the end behavior of the graph of $y = x^{3-2} = x$. So, the graph falls to the left and rises to the right. Construct a table of values consisting of x-values beyond the vertical asymptote.



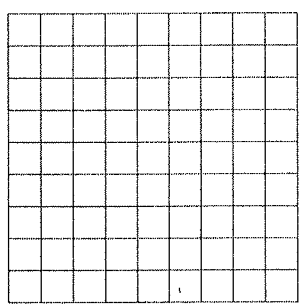
x	-4	-3	-1	1	3	4
y	-5.3	-5.4	0.3	-0.3	5.4	5.3

Plot the points, use the asymptotes, and consider the end behavior to draw the graph.

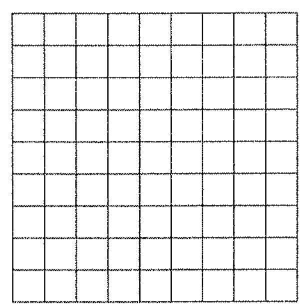
Exercises for Example 3

Graph the function.

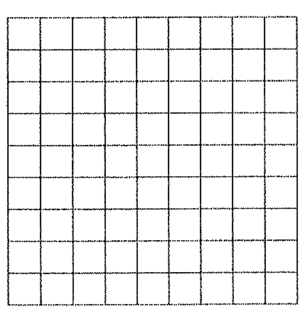
9. $y = \dfrac{-x^2}{x + 1}$



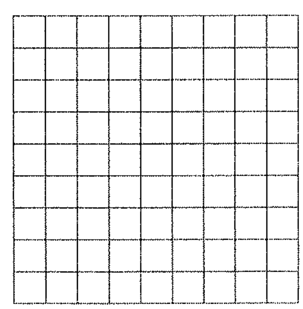
10. $y = \dfrac{x^2 + 1}{x - 2}$



11. $y = \dfrac{x^2 + 3x - 18}{x}$



12. $y = \dfrac{x^2 - 9}{2x}$



Chapter 9

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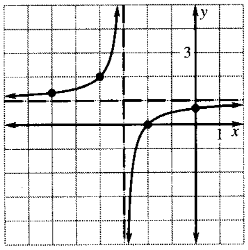
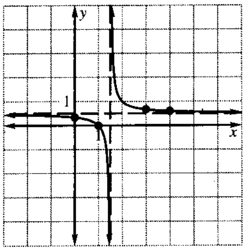
9. domain: all real numbers except -3
range: all real numbers except 1



10. domain: all real numbers except $\frac{3}{2}$
range: all real numbers except $\frac{1}{2}$



11. about \$32

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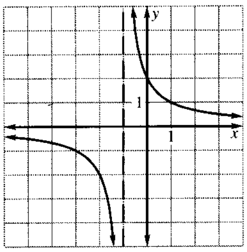

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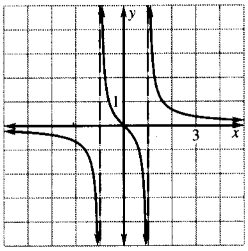

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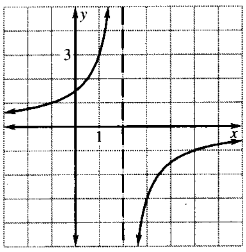

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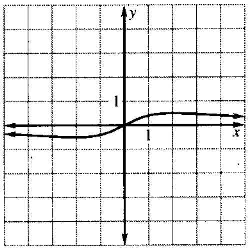

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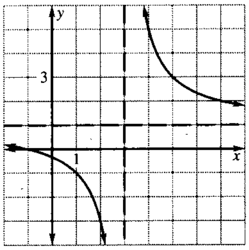

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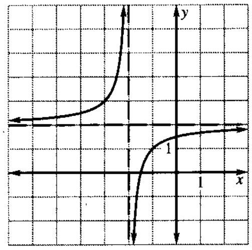

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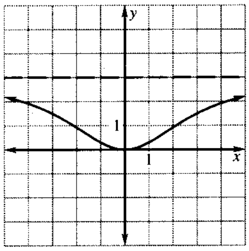

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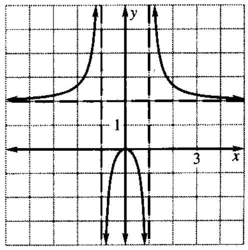

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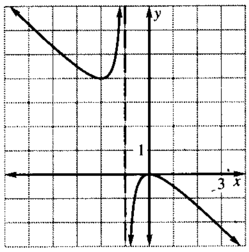

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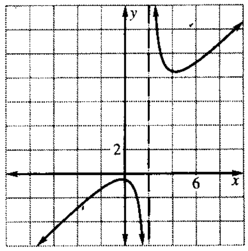

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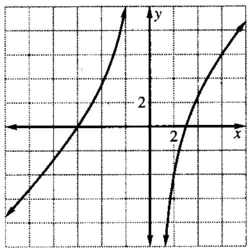

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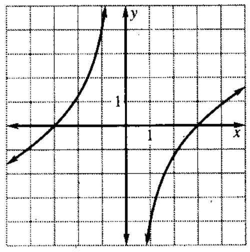

Lesson 9.4

1. $\dfrac{y+9}{2}$ 2. $\dfrac{1}{2}$ 3. $\dfrac{1}{x+3}$ 4. $\dfrac{y}{y-1}$

5. $\dfrac{x(x+3)}{x+1}$ 6. 2 7. -1 8. $\dfrac{20x}{3y^3}$

9. $\dfrac{x}{3(x-1)}$

Lesson 9.5

1. $\dfrac{x^2+6}{2x^2}$ 2. $\dfrac{4x+9}{2x(x+3)}$ 3. $\dfrac{7x+23}{(x+5)(x+1)}$

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$$r(x) = \frac{1}{x^2+3}.$$

The denominator is always greater than 3; it is never 0. We see from Figure 9.31 that r does not have a vertical asymptote.

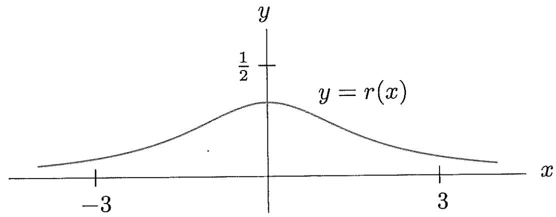


Figure 9.31: The rational function $r(x) = 1/(x^2+3)$ has no vertical asymptote

Exercises and Problems for Section 9.4

Exercises

Are the functions in Exercises 1–6 rational functions? If so, write them in the form $p(x)/q(x)$, the ratio of polynomials.

1. $f(x) = \dfrac{x^2}{2} + \dfrac{1}{x}$

2. $f(x) = \dfrac{\sqrt{x}+1}{x+1}$

3. $f(x) = \dfrac{4^x+3}{3^x-1}$

4. $f(x) = \dfrac{x^2+4}{e^x}$

5. $f(x) = \dfrac{x^2}{x-3} - \dfrac{5}{x-3}$

6. $f(x) = \dfrac{9x-1}{4\sqrt{x}+7} + \dfrac{5x^3}{x^2-1}$

Problems

7. Compare and discuss the long-run behaviors of the following functions:

$$f(x) = \frac{x^2+1}{x^2+5}, \quad g(x) = \frac{x^3+1}{x^2+5}, \quad h(x) = \frac{x+1}{x^2+5}.$$

8. Give examples of rational functions with even symmetry, odd symmetry, and neither. How does the symmetry of $f(x) = p(x)/q(x)$ depend on the symmetry of $p(x)$ and $q(x)$?

Find the horizontal asymptote, if it exists, of the functions in Problems 9–11.

9. $f(x) = \dfrac{1}{1+\dfrac{1}{x}}$

10. $g(x) = \dfrac{(1-x)(2+3x)}{2x^2+1}$

11. $h(x) = 3 - \dfrac{1}{x} + \dfrac{x}{x+1}$

12. Let t be the time in weeks. At time $t = 0$, organic waste is dumped into a pond. The oxygen level in the pond at time t is given by

$$f(t) = \frac{t^2-t+1}{t^2+1}.$$

Assume $f(0) = 1$ is the normal level of oxygen.

(a) Graph this function.
(b) Describe the shape of the graph. What is the significance of the minimum for the pond?
(c) What eventually happens to the oxygen level?
(d) Approximately how many weeks must pass before the oxygen level returns to 75% of its normal level?

13. Find a formula for $f^{-1}(x)$ given that

$$f(x) = \frac{4-3x}{5x-4}.$$

14. Bronze is an alloy, or mixture, of copper and tin. The alloy initially contains 3 kg copper and 9 kg tin. You add x kg of copper to this 12 kg of alloy. The concentration of copper in the alloy is a function of x:

$$f(x) = \text{Concentration of copper} = \frac{\text{Total amount of copper}}{\text{Total amount of alloy}}.$$

(a) Find a formula for f in terms of x, the amount of copper added.

(b) Evaluate the following expressions and explain their significance for the alloy:

(i) $f(\frac{1}{2})$ (ii) $f(0)$ (iii) $f(-1)$
(iv) $f^{-1}(\frac{1}{2})$ (v) $f^{-1}(0)$

(c) Graph $f(x)$ for $-5 \le x \le 5$, $-0.25 \le y \le 0.5$. Interpret the intercepts in the context of the alloy.

(d) Graph $f(x)$ for $-3 \le x \le 100$, $0 \le y \le 1$. Describe the appearance of your graph for large x-values. Does the appearance agree with what you expect to happen when large amounts of copper are added to the alloy?

15. A chemist is studying the properties of a bronze alloy (mixture) of copper and tin. She begins with 2 kg of an alloy that is one-half tin. Keeping the amount of copper constant, she adds small amounts of tin to the alloy. Letting x be the total amount of tin added, define

$$C(x) = \text{Concentration of tin} = \frac{\text{Total amount of tin}}{\text{Total amount of alloy}}.$$

(a) Find a formula for $C(x)$.

(b) Evaluate $C(0.5)$ and $C(-0.5)$. Explain the physical significance of these quantities.

(c) Graph $y = C(x)$, labeling all interesting features. Describe the physical significance of the features you have labeled.

16. The total cost $C(n)$ for a producer to manufacture n units of a good is given by

$$C(n) = 5000 + 50n.$$

The average cost of producing n units is $a(n) = C(n)/n$.

(a) Evaluate and interpret the economic significance of:

(i) $C(1)$ (ii) $C(100)$
(iii) $C(1000)$ (iv) $C(10000)$

(b) Evaluate and interpret the economic significance of:

(i) $a(1)$ (ii) $a(100)$
(iii) $a(1000)$ (iv) $a(10000)$

(c) Based on part (b), what trend do you notice in the values of $a(n)$ as n gets large? Explain this trend in economic terms.

17. Figure 9.32 shows the cost function, $C(n)$, from Problem 16, together with a line, l, that passes through the origin.

(a) What is the slope of line l?

(b) How does line l relate to $a(n_0)$, the average cost of producing n_0 units (as defined in Problem 16)?

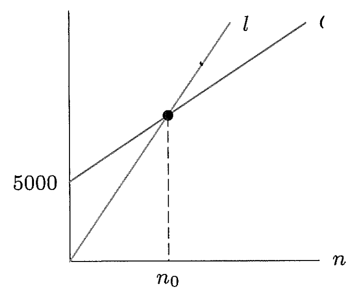


Figure 9.32

18. Typically, the average cost of production (as defined in Problem 16) decreases as the level of production increases. Is this always the case for the goods whose total cost function is graphed in Figure 9.33? Use the result of Problem 17 and explain your reasoning.

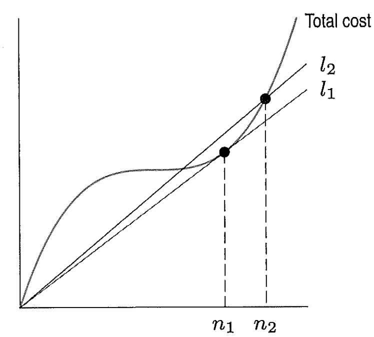


Figure 9.33

19. It costs a company \$30,000 to begin production of a good, plus \$3 for every unit of the good produced. Let x be the number of units produced by the company.

(a) Find a formula for $C(x)$, the total cost for the production of x units of the good.

(b) Find a formula for the company's average cost per unit, $a(x)$.

(c) Graph $y = a(x)$ for $0 < x \le 50{,}000$, $0 \le y \le 10$. Label the horizontal asymptote.

(d) Explain in economic terms why the graph of a has the long-run behavior that it does.

(e) Explain in economic terms why the graph of a has the vertical asymptote that it does.

(f) Find a formula for $a^{-1}(y)$. Give an economic interpretation of $a^{-1}(y)$.

(g) The company makes a profit if the average cost of its good is less than \$5 per unit. Find the minimum number of units the company can produce and make a profit.

When Numerator and Denominator Have the Same Zeros: Holes

The rational function $h(x) = \dfrac{x^2+x-2}{x-1}$ is undefined at $x=1$ because the denominator equals zero at $x=1$. However, the graph of h does not have a vertical asymptote at $x=1$ because the numerator of h also equals zero at $x=1$. At $x=1$,

$$h(1) = \frac{x^2+x-2}{x-1} = \frac{1^2+1-2}{1-1} = \frac{0}{0},$$

and this ratio is undefined. What does the graph of h look like? Factoring the numerator of h gives

$$h(x) = \frac{(x-1)(x+2)}{x-1} = \frac{x-1}{x-1}(x+2).$$

For any $x \neq 1$, we can cancel $(x-1)$ top and bottom and rewrite the formula for h as

$$h(x) = x+2, \qquad \text{provided } x \neq 1.$$

Thus, the graph of h is the line $y=x+2$ except at $x=1$, where h is undefined. The line $y=x+2$ contains the point $(1,3)$, but that the graph of h does not. Therefore, we say that the graph of h has a *hole* in it at the point $(1,3)$. See Figure 9.39.

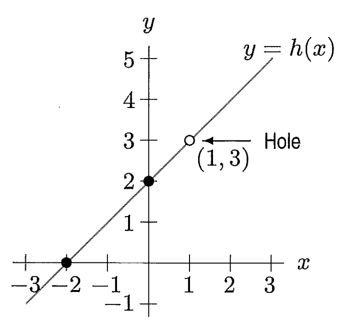


Figure 9.39: The graph of $y=h(x)$ is the line $y=x+2$, except at the point $(1,3)$, where it has a hole

Exercises and Problems for Section 9.5

Exercises

In Exercises 1–4, what are the x-intercepts, y-intercepts, and horizontal and vertical asymptotes (if any)?

1. $f(x) = \dfrac{x-2}{x-4}$

2. $g(x) = \dfrac{x^2-9}{x^2+9}$

3. $h(x) = \dfrac{x^2-4}{x^3+4x^2}$

4. $k(x) = \dfrac{x(4-x)}{x^2-6x+5}$

For the rational functions in Exercises 5–8, find all zeros and vertical asymptotes and describe the long-run behavior. Then graph the function without a calculator.

5. $y = \dfrac{x+3}{x+5}$

6. $y = \dfrac{x+3}{(x+5)^2}$

7. $y = \dfrac{x-4}{x^2-9}$

8. $y = \dfrac{x^2-4}{x-9}$

Graph the functions in Exercises 9–10 without a calculator.

9. $y = 2 + \dfrac{1}{x}$

10. $y = \dfrac{2x^2-10x+12}{x^2-16}$

11. Let $f(x) = \dfrac{1}{x-3}$.

(a) Complete Table 9.8 for x-values close to 3. What happens to the values of $f(x)$ as x approaches 3 from the left? From the right?

Table 9.8

x	2	2.9	2.99	3	3.01	3.1	4
$f(x)$							

(b) Complete Tables 9.9 and 9.10. What happens to the values of $f(x)$ as x takes very large positive values? As x takes very large negative values?

Table 9.9

x	5	10	100	1000
$f(x)$				

Table 9.10

x	-5	-10	-100	-1000
$f(x)$				

(c) Without a calculator, graph $y = f(x)$. Give equations for the horizontal and vertical asymptotes.

12. Let $g(x) = \dfrac{1}{(x+2)^2}$.

(a) Complete Table 9.11 for x-values close to -2. What happens to the values of $g(x)$ as x approaches -2 from the left? From the right?

Table 9.11

x	-3	-2.1	-2.01	-2	-1.99	-1.9	-1
$g(x)$							

(b) Complete Tables 9.12 and 9.13. What happens to the values of $g(x)$ as x takes very large positive values? As x takes very large negative values?

Table 9.12

x	5	10	100	1000
$g(x)$				

Table 9.13

x	-5	-10	-100	-1000
$g(x)$				

(c) Without a calculator, graph $y = g(x)$. Give equations for the horizontal and vertical asymptotes.

13. Let $F(x) = \dfrac{x^2-1}{x^2}$.

(a) Complete Table 9.14 for x-values close to 0. What happens to the values of $F(x)$ as x approaches 0 from the left? From the right?

Table 9.14

x	-1	-0.1	-0.01	0	0.01	0.1	1
$F(x)$							

(b) Complete Tables 9.15 and 9.16. What happens to the values of $F(x)$ as x takes very large positive values? As x takes very large negative values?

Table 9.15

x	5	10	100	1000
$F(x)$				

Table 9.16

x	-5	-10	-100	-1000
$F(x)$				

(c) Without a calculator, graph $y = F(x)$. Give equation for the horizontal and vertical asymptotes.

14. Let $G(x) = \dfrac{2x}{x+4}$.

(a) Complete Table 9.17 for x-values close to -4. What happens to the values of $G(x)$ as x approaches -4 from the left? From the right?

Table 9.17

x	-5	-4.1	-4.01	-4	-3.99	-3.9	-3
$G(x)$							

(b) Complete Tables 9.18 and 9.19. What happens to the values of $G(x)$ as x takes very large positive values? As x takes very large negative values?

Table 9.18

x	5	10	100	1000
$G(x)$				

Table 9.19

x	-5	-10	-100	-1000
$G(x)$				

(c) Without a calculator, graph $y = G(x)$. Give equations for the horizontal and vertical asymptotes.

Problems

15. Let $f(x) = x^2 + 5x + 6$ and $g(x) = x^2 + 1$.

(a) What are the zeros of f and g?

(b) Let $r(x) = f(x)/g(x)$. Graph r. Does r have zeros? Vertical asymptotes? What is its long-run behavior as $x \to \pm\infty$?

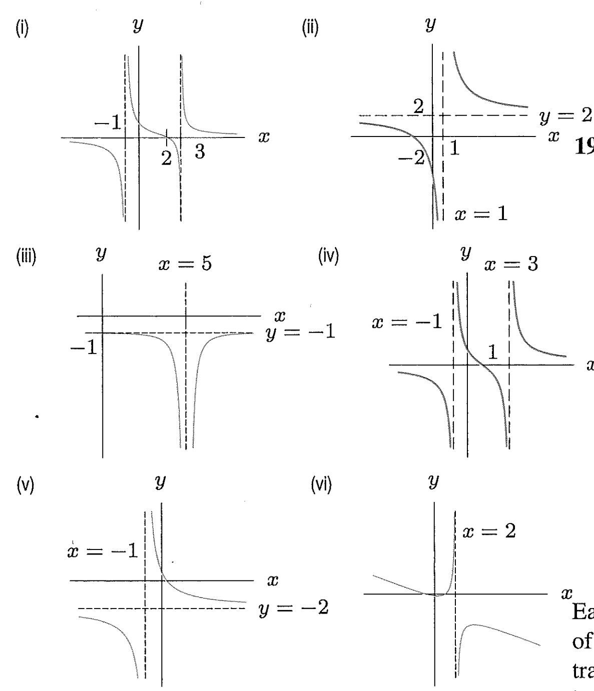
(c) Let $s(x) = g(x)/f(x)$. If you graph s in the window $-10 \le x \le 10$, $-10 \le y \le 10$, it appears to have a zero near the origin. Does it? Does s have a vertical asymptote? What is its long-run behavior?

16. Without a calculator, match the functions (a)–(f) with their graphs in (i)–(vi) by finding the zeros, asymptotes, and end behavior for each function.

(a) $y = \dfrac{-1}{(x-5)^2} - 1$ **(b)** $y = \dfrac{x-2}{(x+1)(x-3)}$

(c) $y = \dfrac{2x+4}{x-1}$ **(d)** $y = \dfrac{1}{x+1} + \dfrac{1}{x-3}$

(e) $y = \dfrac{1-x^2}{x-2}$ **(f)** $y = \dfrac{1-4x}{2x+2}$



17. Suppose that n is a constant and that $f(x)$ is a function defined when $x = n$. Complete the following sentences.

(a) If $f(n)$ is large, then $\dfrac{1}{f(n)}$ is . . .

(b) If $f(n)$ is small, then $\dfrac{1}{f(n)}$ is . . .

(c) If $f(n) = 0$, then $\dfrac{1}{f(n)}$ is . . .

(d) If $f(n)$ is positive, then $\dfrac{1}{f(n)}$ is . . .

(e) If $f(n)$ is negative, then $\dfrac{1}{f(n)}$ is . . .

18. **(a)** Use the results of Problem 17 to graph $y = 1/f(x)$ given the graph of $y = f(x)$ in Figure 9.40.

(b) Find a possible formula for the function in Figure 9.40. Use this formula to check your graph for part (a).

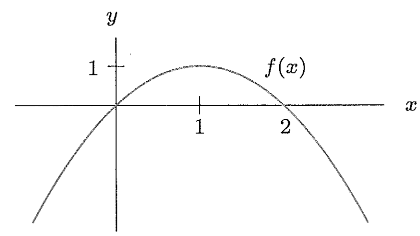


Figure 9.40

19. Use the graph of f in Figure 9.41 to graph

(a) $y = -f(-x) + 2$ (b) $y = \dfrac{1}{f(x)}$

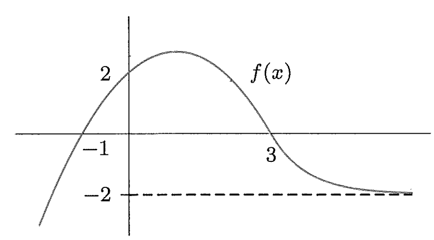


Figure 9.41

Each of the functions in Problems 20–22 is a transformation of $y = 1/x^p$. For each function, determine p, describe the transformation in words, and graph the function and label any intercepts and asymptotes.

20. $f(x) = \dfrac{1}{x-3} + 4$

21. $g(x) = -\dfrac{1}{(x-2)^2} - 3$

22. $h(x) = \dfrac{1}{x-1} + \dfrac{2}{1-x} + 2$

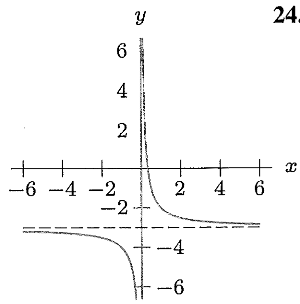
Problems 23–25 show a translation of $y = 1/x$.

(a) Find a possible formula for the graph.

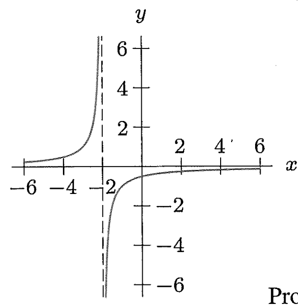
(b) Write the formula from part (a) as the ratio of two linear polynomials.

(c) Find the coordinates of the intercepts of the graph.

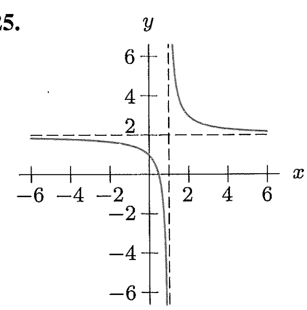
23.



24.



25.



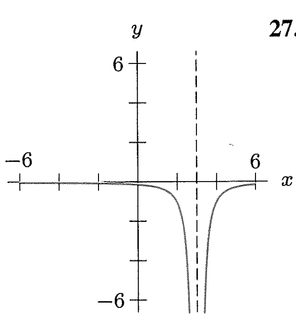
Problems 26–28 show a translation of $y = 1/x^2$.

(a) Find a formula for the graph.

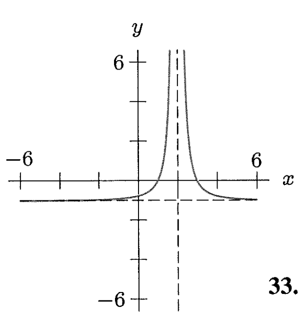
(b) Write the formula from part (a) as the ratio of two polynomials.

(c) Find the coordinates of any intercepts of the graph.

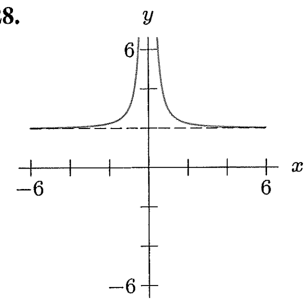
26.



27.



28.



Problems 29–32 give values of translations of either $y = 1/x$ or $y = 1/x^2$. In each case

(a) Determine if the values are from a translation of $y = 1/x$ or $y = 1/x^2$. Explain your reasoning.

(b) Find a possible formula for the function.

29.

x	y
2.7	12.1
2.9	101
2.95	401
3	Undefined
3.05	401
3.1	101
3.3	12.1

30.

x	y
−1000	0.499
−100	0.490
−10	0.400
10	0.600
100	0.510
1000	0.501

31.

x	y
−1000	1.000001
−100	1.00001
−10	1.01
10	1.01
100	1.0001
1000	1.000001

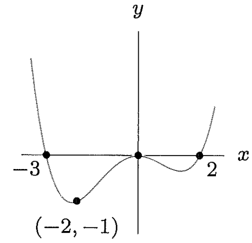
32.

x	y
1.5	−1.5
1.9	−9.5
1.95	−19.5
2	Undefined
2.05	20.5
2.1	10.5
2.5	2.5

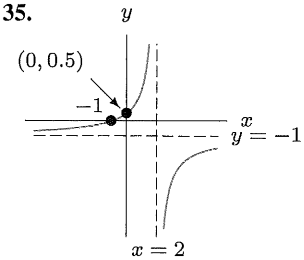
33. Cut four equal squares from the corners of a $8.5'' \times 11''$ piece of paper. Fold up the sides to create an open box. Find the dimensions of the box with the maximum volume per surface area.

Find possible formulas for the functions in Problems 34–41.

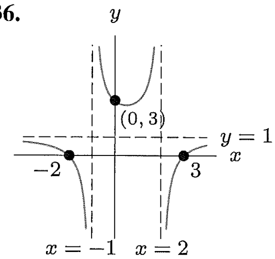
34.



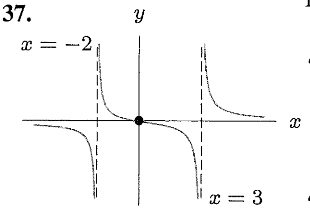
35.



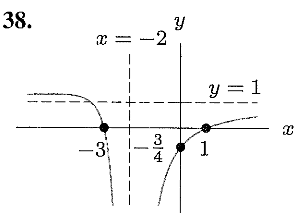
36.



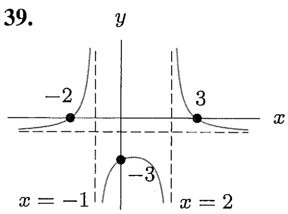
37.



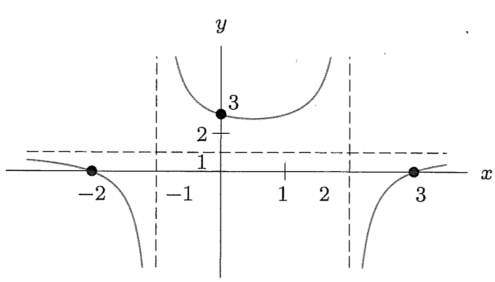
38.



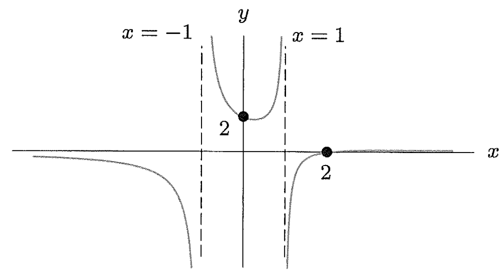
39.



40.



41.



In Problems 42–44, find a possible formula for the rational functions.

42. The graph of $y = f(x)$ has one vertical asymptote, at $x = -1$, and a horizontal asymptote at $y = 1$. The graph of f crosses the y-axis at $y = 3$ and crosses the x-axis once, at $x = -3$.

43. The graph of $y = g(x)$ has two vertical asymptotes: one at $x = -2$ and one at $x = 3$. It has a horizontal asymptote of $y = 0$. The graph of g crosses the x-axis once, at $x = 5$.

44. The graph of $y = h(x)$ has two vertical asymptotes: one at $x = -2$ and one at $x = 3$. It has a horizontal asymptote of $y = 1$. The graph of h touches the x-axis once, at $x = 5$.

45. The graph of $f(x) = \dfrac{18 - 11x + x^2}{x - 2}$ is a line with a hole in it. What is the equation of the line? What are the coordinates of the hole?

46. The graph of $g(x) = \dfrac{x^3 + 5x^2 + x + 5}{x + 5}$ is a parabola with a hole in it. What is the equation of the parabola? What are the coordinates of the hole?

47. Write a formula for a function, $h(x)$, whose graph is identical to the graph of $y = x^3$, except that the graph of h has a hole at $(2, 8)$. Express the formula as a ratio of two polynomials

9.6 COMPARING POWER, EXPONENTIAL, AND LOG FUNCTIONS

In preceding chapters, we encountered exponential, power, and logarithmic functions. In this section, we compare the long and short-run behaviors of these functions.

Comparing Power Functions

For power functions $y = kx^p$ for large x, the higher the power of x, the faster the function climbs. See Figure 9.42. Not only are the higher powers larger, but they are *much* larger. This is because if $x = 100$, for example, 100^5 is one hundred times as big as 100^4, which is one hundred times as big as 100^3. As x gets larger (written as $x \to \infty$), any positive power of x grows much faster than all lower powers of x. We say that, as $x \to \infty$, higher powers of x *dominate* lower powers.

Section 9.4

1 Rational; $(x^3+2)/(2x)$

3 Not rational

5 Rational; $(x^2-5)/(x-3)$

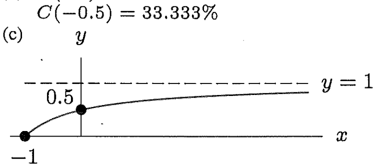
7 As $x \to \pm\infty$, $f(x) \to 1$, $g(x) \to x$, and $h(x) \to 0$

9 $y = 1$

11 $y = 4$

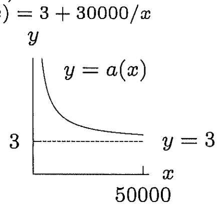
13 $f^{-1}(x) = (4x+4)/(5x+3)$

15 (a) $C(x) = (1+x)/(2+x)$
(b) $C(0.5) = 60\%$;
$C(-0.5) = 33.333\%$
(c)



17 (a) $C(n_0)/n_0$
(b) Slope is average cost for n_0 units

19 (a) $C(x) = 30000 + 3x$
(b) $a(x) = 3 + 30000/x$
(c)



(f) $a^{-1}(y) = 30000/(y-3)$
(g) 15,000

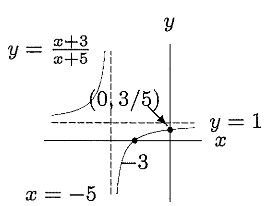
21 (a) $h(x) = (2x^3+2)/(3x^2)$

Section 9.5

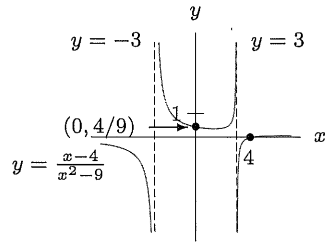
1 x-int: $x = 2$
y-int: $y = 1/2$
Horiz asy: $y = 1$
Vert asy: $x = 4$

3 x-int: $x = \pm 2$
y-int: None
Horiz asy: $y = 0$
Vert asy: $x = 0, x = -4$

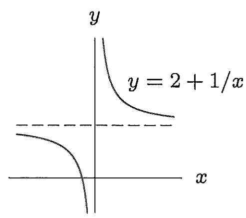
5 Zero: $x = -3$;
Asymptote: $x = -5$;
$y \to 1$ as $x \to \pm\infty$



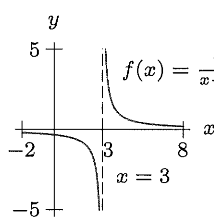
7 Zeros: $x = 4$;
Asymptote: $x = \pm 3$;
$y \to 0$ as $x \to \pm\infty$



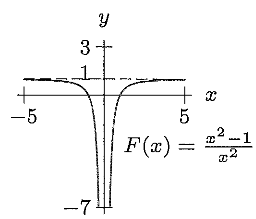
9



11 (c) Horizontal: $y = 0$
Vertical: $x = 3$



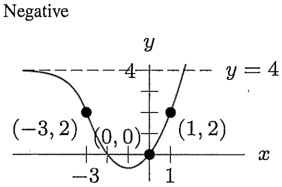
13 (c) Horizontal: $y = 1$
Vertical: $x = 0$



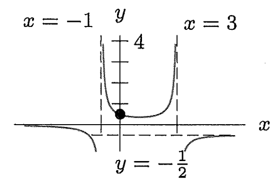
15 (a) $-2, -3$; None
(b) $-2, -3$; No; $r(x) \to 1$ as $x \to \pm\infty$
(c) No; Yes at $x = -2$ and $x = 3$;
$s(x) \to 1$ as $x \to \pm\infty$

17 (a) Small
(b) Large
(c) Undefined
(d) Positive
(e) Negative

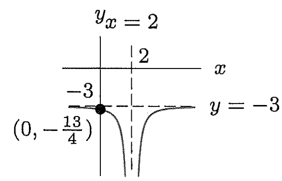
19 (a)



(b)



21 $p = 2$, $(0, -13/4)$
$x = 2, y = -3$



23 (a) $y = (1/x) - 3$
(b) $y = (-3x+1)/x$
(c) $(1/3, 0)$

25 (a) $y = 1/(x-1) + 2$
(b) $y = (2x-1)/(x-1)$
(c) $(1/2, 0)$ and $(0, 1)$

27 (a) $y = 1/(x-2)^2 - 1$
(b) $y =$
$(-x^2+4x-3)/(x^2-4x+4)$
(c) $(0, -3/4)$, $(1, 0)$ and $(3, 0)$

29 (a) $1/x^2$
(b) $y = \dfrac{1}{(x-3)^2} + 1$

31 (a) $1/x^2$
(b) $y = \dfrac{1}{x^2} + 1$

33 7.123 by 4.623 by 1.939 inches

35 $y = -(x+1)/(x-2)$

37 $y = x/((x+2)(x-3))$

39 $y = -(x-3)(x+2)/((x+1)(x-2))$

41 $y = (x-2)/((x+1)(x-1))$

43 $g(x) = (x-5)/((x+2)(x-3))$

45 $y = x - 9$; $(2, -7)$

47 $h(x) = (x^4 - 2x^3)/(x-2)$

Section 9.6

1 (a) Neither
(b) $n(x) = 6 \cdot 8^x$
(c) $p(x) = 25^x$
(d) Neither
(e) $r(x) = 2(\frac{1}{9})^x$
(f) $s(x) = \frac{4}{5}x^3$

5 $y = 50x^{1.1}$

7 $y = 6x^{35}$

9 3^{-x} approaches zero faster

11 (a) $f(x) = 720x - 702$
(b) $f(x) = 2(9)^x$
(c) $f(x) = 18x^4$

Sheet 1241B. Rational and Hyperbolic Function Graphs

RATIONAL FUNCTIONS

$$f(x) = \frac{p(x)}{q(x)}$$

$p(x)$ and $q(x)$ are polynomials.

KNOW HOW TO FIND VERTICAL + HORIZONTAL ASYMPTOTES, ZEROS (X-INT.) AND Y-INTERCEPTS.

- UNIT HYPERBOLIC FUNCTION

$$f(x) = \frac{1}{x}$$

HYPERBOLIC FUNCTION Centered at (H, K)

$$g(x) = \frac{a}{x-H} + K$$

- COMMON RATIONAL FUNCTION

$$h(x) = \frac{1}{x^2}$$

$$m(x) = \frac{1}{(x-H)^2} + K$$

YOU NEED TO BE ABLE TO GRAPH EQUATIONS BY HAND AND WITH TI-83 CALCULATOR AND FIND EQUATIONS FROM GRAPHS.

SHEET #161: Sheet 1246

RATIONAL FUNCTION GRAPHS

NAME: ______________ Period: ____

GROUP MEMBERS: ______________ (Version 3)

____

FOR QUESTIONS 1 AND 2,
SKETCH THESE FUNCTIONS, $f(x)$,
BUT FIRST, FIND (if they exist)

a, ZEROS

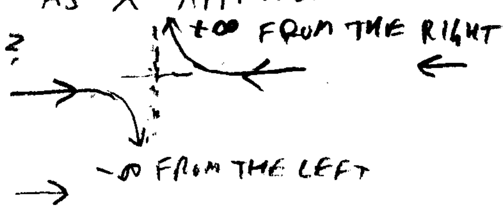
b, Y-INTERCEPT

c, VERTICAL ASYMPTOTES

d, HORIZONTAL ASYMPTOTES

e, SLANT ASYMPTOTES

f, THE BEHAVIOR NEAR VERTICAL ASYMPTOTES. DOES $f(x)$ APPROACH $+\infty$ OR $-\infty$ AS X APPROACHES THE ASYMPTOTE?
SEE EXAMPLE:



g, HOLES

Start answering the questions with a=1.
Check with [Zoom] 4:ZDecimal

1. $f(x) = \dfrac{x-a}{x-2a}$ $(a>0)$

2. $f(x) = \dfrac{x^2-a^2}{x-a}$ $(a>0)$

SHEET #161: Sheet 1246 Solutions

RATIONAL FUNCTION GRAPHS

NAME: $\boxed{\text{KEY}}$ Period: ____

GROUP MEMBERS: ______ (Version 3.)

FOR QUESTIONS 1 AND 2,
SKETCH THESE FUNCTIONS, $f(x)$,
BUT FIRST, FIND (if they exist)

a, Zeros

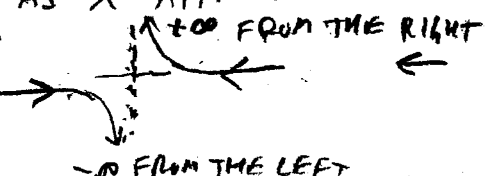
b, Y-INTERCEPT

c, VERTICAL ASYMPTOTES

d, HORIZONTAL ASYMPTOTES

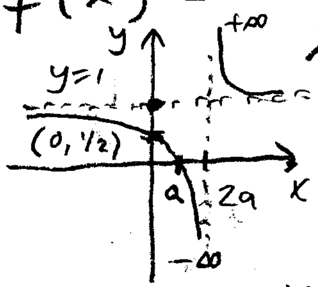
e, SLANT ASYMPTOTES

f, THE BEHAVIOR NEAR VERTICAL ASYMPTOTES. DOES $f(x)$ APPROACH $+\infty$ OR $-\infty$ AS X APPROACHES THE ASYMPTOTE?
SEE EXAMPLE:

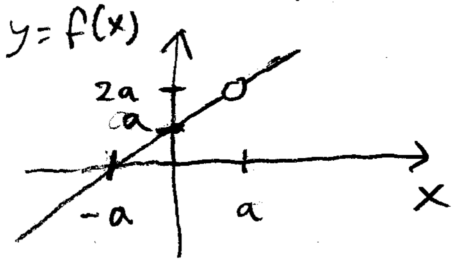


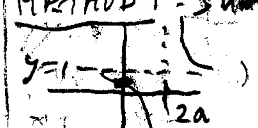
g, HOLES

1. $f(x) = \dfrac{x-a}{x-2a}$ $(a>0)$



2. $f(x) = \dfrac{x^2-a^2}{x-a}$ $(a>0)$



1. $f(x)=\dfrac{x-a}{x-2a}$	2. $f(x)=\dfrac{x^2-a^2}{x-a}$
$x-a=0$ $\boxed{x=a}$	$x^2-a^2=0$ $(x-a)(x+a)=0$ $\boxed{x=-a}$ +SEE BELOW
$f(0)=\dfrac{-a}{-2a}=\boxed{\dfrac{1}{2}}$	$f(0)=\dfrac{-a^2}{-a}=\boxed{a}$
$x-2a=0$ $\boxed{x=2a}$ $x\to\infty$ $f(x)\to\dfrac{x}{x}\to 1$ $\boxed{y=1}$ $\boxed{\text{NO SLANT}}$	$f(x)=\dfrac{(x-a)(x+a)}{x-a}$ $=x+a,\ x\neq a$ $\boxed{\text{NO ASYMPTOTES}}$ DIVIDE BY 0 FOR $x=a$ $\downarrow$ (SO NO ZERO AT $x=a$)
METHOD 1: SKETCH  $f(0)=1/2$ SAYS $x\to 2a^-$ (FROM LEFT) $f(x)\to-\infty$ $x\to 2a^+$ (FROM RIGHT) $f(x)\to\infty$	HOLE AT $x=a$ $\boxed{(a, 2a)}$
METHOD 2: PLUG IN VALUES SUCH AS $x=1.9a$ AND $x=2.1a$ $f(1.9a)=[\text{Left}]$ $\dfrac{1.9a-a}{1.9a-2a}=$ $=\dfrac{0.9a}{-0.1a}=-9a$ $\boxed{f\to-\infty}$ as $x\to 2a^-$ $f(2.1a)=[\text{Right}]$ $=\dfrac{2.1a-a}{2.1a-2a}=$ $=\dfrac{1.1a}{0.1a}=11a$ $\boxed{f\to\infty}$ as $x\to 2a^+$.	

~~SHEET~~ #162 : Sheet 1247

RATIONAL FUNCTIONS:

3 CASES OF END BEHAVIOR (= LONG RUN BEHAVIOR)

including HORIZONTAL AND OTHER ASYMPTOTES.

Let $p(x) = a_n x^n + \cdots + a_1 x + a_0$

and $q(x) = b_m x^m + \cdots + b_1 x + b_0$

$f(x) = \frac{p(x)}{q(x)}$ = RATIO OF POLYNOMIALS = $\boxed{\text{RATIONAL FUNCTION}}$

$a_n x^n$ is leading term of $p(x)$

$b_m x^m$ is leading term of $q(x)$

n is degree of $p(x)$. a_n is leading coefficient of $p(x)$.

m is degree of $q(x)$ b_m is leading coefficient of $q(x)$.

GENERAL METHOD

DIVIDE LEADING TERMS = $\frac{a_n x^n}{b_m x^m}$

AND TAKE LIMIT AS $x \to \pm\infty$

TASK: FIND END BEHAVIOR AND/OR HORIZONTAL Asymptotes for $y = f(x) = p(x)/q(x)$.

CASE 1. $n = m$. Degree of $p(x)$ = Degree of $q(x)$

EX. $f(x) = \frac{3x^3 + 2x + 7}{4x^3 + 8}$

END BEHAVIOR FUNCTION: $y = \frac{3x^3}{4x^3} = \frac{3}{4}$. [LIMIT IS A NUMBER = 3/4.]

Horizontal Asymptote $\boxed{y = \frac{3}{4}}$ [THE EQUATION OF A FUNCTION.]

CASE 2. $n > m$ Degree of $p(x)$ = Degree of $q(x)$

EX. $f(x) = \frac{3x^4 + 2x + 7}{4x^3 + 8}$

END BEHAVIOR FUNCTION: $y = \frac{3x^4}{4x^3}$ $\boxed{y = \frac{3}{4}x}$

SINCE $y = \frac{3}{4}x$ IS A LINE IT IS CALLED A "SLANT" ASYMPTOTE*

THE LIMIT IS $y \to \boxed{\pm\infty}$ (NO HORIZONTAL ASYMPTOTE)

CASE 3. $n < m$ Degree of $p(x)$ < Degree of $q(x)$

EX. $f(x) = \frac{3x^3 + 2x + 7}{4x^4 + 8}$

END BEHAVIOR FUNCTION: $y = \frac{3x^3}{4x^4}$. $\boxed{y = \frac{3}{4}\frac{1}{x}}$

Horizontal Asymptote $\boxed{y = 0}$ [because $\frac{1}{\text{BIG \#}}$ = SMALL #]

* IF END BEHAVIOR FUNCTION IS NOT A LINE, IT IS USUALLY NOT CALLED AN ASYMPTOTE. Ex. $f(x) = \frac{3x^5 + 2x + 7}{4x^3 + 8}$ has...

(ALTHOUGH IT COULD BE CALLED "NONLINEAR" ASYMPTOTE.)

... this END BEHAVIOR FUNCTION: $y = \frac{3x^5}{4x^3}$. $\boxed{y = \frac{3}{4}x^2}$

NOTE THAT ALL END BEHAVIOR FUNCTIONS ARE PROPORTIONAL TO THE RATIO OF LEADING COEFFICIENTS (except $y=0$) $y = \frac{3}{4}\frac{1}{x}$, $y = \frac{3}{4}$, $y = \frac{3}{4}x$, $y = \frac{3}{4}x^2$.