Name: ____ Block/Period: _____

Version 3

Matrix Packet

with systems of equations

THIS UNIVERSE IS NOT WHAT IT APPEARS

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

YOU CAN SENSE IT
WHEN YOU LOOK INTO YOUR INNER PRODUCT

$$\begin{bmatrix} f(x) \end{bmatrix} \quad \begin{bmatrix} \frac{\delta}{\delta x} \\ \frac{\delta}{\delta y} \\ \frac{\delta}{\delta z} \end{bmatrix}$$

THERE IS ANOTHER ECHELON OF REALITY
A COMPLEX SPACE WHERE THE TRUTH LIES

A TRUE VECTOR SPACE
WHICH WILL TRANSFORM YOUR IDENTITY

YOUR FUTURE IS NOT DETERMINATE
YOU HAVE THE CHOICE

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

TO ENTER

$$\begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

[THE MATRIX]

NAME ______________________ DATE __________

Practice with Examples

For use with pages 139–145

GOAL Graph and solve systems of linear equations in two variables

VOCABULARY

A **system of two linear equations** in two variables x and y consists of two equations, $Ax + By = C$ and $Dx + Ey = F$.

A **solution** of a system of linear equations in two variables is an ordered pair (x, y) that satisfies *both* equations.

EXAMPLE 1 *Solving a System Graphically*

Solve the system.

$x - y = 1$ **Equation 1**

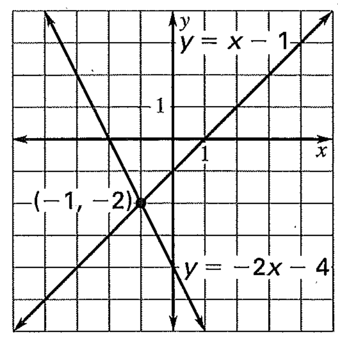
$2x + y = -4$ **Equation 2**

SOLUTION

Begin by writing each equation in slope-intercept form.

$y = x - 1$ **Equation 1**

$y = -2x - 4$ **Equation 2**



Then sketch the graph of each equation. From the graph, the lines appear to intersect at $(-1, -2)$. To algebraically check this, substitute -1 for x and -2 for y into each of the original equations.

$-1 - (-2) = 1$ ✓ Equation 1 checks.

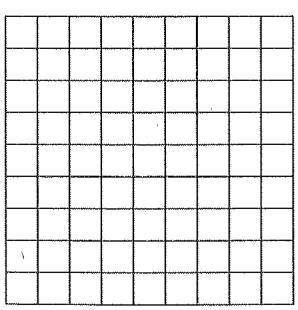
$2(-1) + (-2) = -4$ ✓ Equation 2 checks.

The solution is $(-1, -2)$.

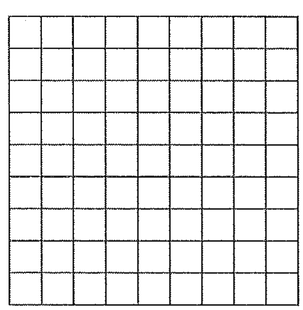
Exercises for Example 2

Graph the linear system and estimate the solution. Then check the solution algebraically.

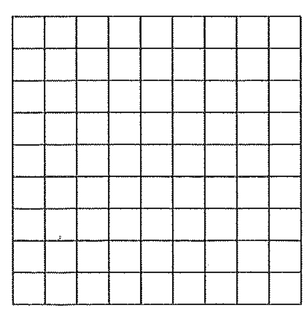
1. $x - 2y = 4$
$y = -x + 4$



2. $y = -3x + 6$
$2x + 3y = -3$



3. $2x - y = -4$
$y = 3$



LESSON 3.2

NAME ______________________ DATE __________

Practice with Examples

For use with pages 148–155

GOAL **Use algebraic methods to solve linear systems**

VOCABULARY

Two methods for solving linear systems are given below.

The Substitution Method

Step 1: Solve one of the equations for one of its variables.

Step 2: Substitute the expression from Step 1 into the other equation and solve for the other variable.

Step 3: Substitute the value from Step 2 into the revised equation from Step 1 and solve.

The Linear Combination Method

Step 1: Multiply one or both of the equations by a constant to obtain coefficients that differ only in sign for one of the variables.

Step 2: Add the revised equations from Step 1. Combining like terms will eliminate one of the variables. Solve for the remaining variable.

Step 3: Substitute the value obtained in Step 2 into either of the original equations and solve for the other variable.

EXAMPLE 1 *The Substitution Method*

Use the substitution method to solve the linear system.

$6x + y = -2$ **Equation 1**

$4x - 3y = 17$ **Equation 2**

SOLUTION

Solve Equation 1 for y.

$6x + y = -2$ Write Equation 1.

$y = -6x - 2$ Revised Equation 1

Substitute $-6x - 2$ for y in Equation 2 and solve for x.

$4x - 3y = 17$ Write Equation 2.

$4x - 3(-6x - 2) = 17$ Substitute $-6x - 2$ for y.

$4x + 18x + 6 = 17$ Distributive property

$x = \frac{1}{2}$ Solve for x.

Substitute the value of x into the revised Equation 1 and solve for y.

$y = -6x - 2$ Revised Equation 1

$y = -6\left(\frac{1}{2}\right) - 2$ Substitute $\frac{1}{2}$ for x.

$y = -5$ Solve for y.

The solution is $\left(\frac{1}{2}, -5\right)$.

Chapter 3

NAME ______________________ DATE __________

Practice with Examples

For use with pages 148–155

Exercises for Example 1

Solve the linear system using the substitution method.

1. $2x - y = 6$
   $2x + 2y = -9$

2. $2x + 3y = 7$
   $x - 2y = -7$

3. $-2x + y = 0$
   $-x + y = 2$

4. $2x - 5y = 9$
   $y = 3x - 7$

5. $-3x + 4y = 1$
   $x = 2y + 1$

6. $6x + 2y = 11$
   $y = -4x + 6$

LESSON 3.2 CONTINUED

NAME ______________________________ DATE ____________

Practice with Examples

For use with pages 148–155

EXAMPLE 2 The Linear Combination Method

Use the linear combination method to solve the linear system.

$6x + 3y = 3$ **Equation 1**

$8x + 4y = 4$ **Equation 2**

SOLUTION

$24x + 12y = 12$ Multiply Equation 1 by 4.

$-24x - 12y = -12$ Multiply Equation 2 by -3, so the x-coefficients differ only in sign.

$0 = 0$ Add the equations.

Because the statement $0 = 0$ is always true, there are infinitely many solutions.

Exercises for Example 2

Solve the linear system using the linear combination method.

7. $9x + 2y = 0$
$3x - 5y = 17$

8. $2x - 3y = 5$
$-6x + 9y = 12$

9. $4x + 5y = 13$
$3x + y = -4$

10. $2x - 5y = -4$
$4x + 3y = 5$

11. $11x + 6y = 1$
$3x + 2y = -3$

12. $6x - 3y = -3$
$8x - 4y = -4$

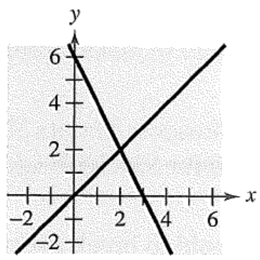
Chapter 3

Sheet 322: Graphing and Solving Systems

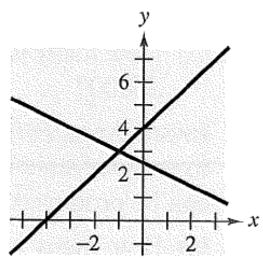
PC 5th Ch. 7–1, p. 551.

In Exercises 5–14, solve the system by the method of substitution. Check your solution graphically.

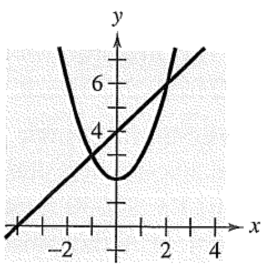
5. $\begin{cases} 2x + y = 6 \\ -x + y = 0 \end{cases}$



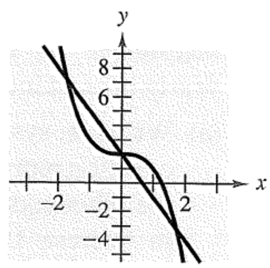
6. $\begin{cases} x - y = -4 \\ x + 2y = 5 \end{cases}$



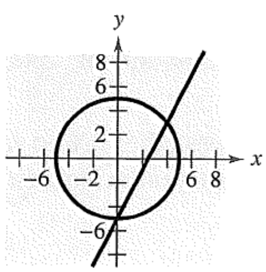
7. $\begin{cases} x - y = -4 \\ x^2 - y = -2 \end{cases}$



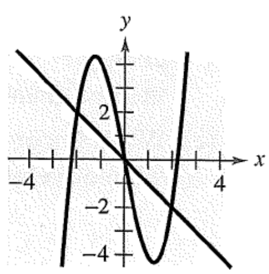
8. $\begin{cases} 3x + y = 2 \\ x^3 - 2 + y = 0 \end{cases}$



9. $\begin{cases} -2x + y = -5 \\ x^2 + y^2 = 25 \end{cases}$



10. $\begin{cases} x + y = 0 \\ x^3 - 5x - y = 0 \end{cases}$



5. (2, 2) **7.** (2, 6), (−1, 3) **9.** (0, −5), (4, 3)

In Exercises 43–50, use a graphing utility to approximate all points of intersection of the graphs.

43. $\begin{cases} y = e^x \\ x - y + 1 = 0 \end{cases}$ **44.** $\begin{cases} y = -4e^{-x} \\ y + 3x + 8 = 0 \end{cases}$

45. $\begin{cases} x + 2y = 8 \\ y = \log_2 x \end{cases}$

46. $\begin{cases} y = -2 + \ln(x - 1) \\ 3y + 2x = 9 \end{cases}$

47. $\begin{cases} y = \sqrt{x} \\ y = x \end{cases}$ **48.** $\begin{cases} x - y = 3 \\ x - y^2 = 1 \end{cases}$

49. $\begin{cases} x^2 + y^2 = 169 \\ x^2 - 8y = 104 \end{cases}$ **50.** $\begin{cases} x^2 + y^2 = 4 \\ 2x^2 - y = 2 \end{cases}$

In Exercises 51–62, solve the system graphically or algebraically. Explain your choice of method.

51. $\begin{cases} y = 2x \\ y = x^2 + 1 \end{cases}$ **52.** $\begin{cases} x + y = 4 \\ x^2 + y = 2 \end{cases}$

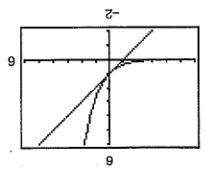
53. $\begin{cases} 3x - 7y + 6 = 0 \\ x^2 - y^2 = 4 \end{cases}$ **54.** $\begin{cases} x^2 + y^2 = 25 \\ 2x + y = 10 \end{cases}$

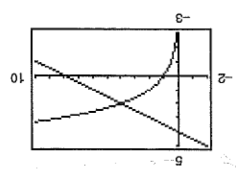
55. $\begin{cases} x - 2y = 4 \\ x^2 - y = 0 \end{cases}$ **56.** $\begin{cases} y = (x + 1)^3 \\ y = \sqrt{x - 1} \end{cases}$

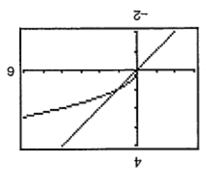
57. $\begin{cases} y - e^{-x} = 1 \\ y - \ln x = 3 \end{cases}$ **58.** $\begin{cases} x^2 + y = 4 \\ e^x - y = 0 \end{cases}$

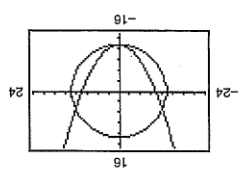
59. $\begin{cases} y = x^4 - 2x^2 + 1 \\ y = 1 - x^2 \end{cases}$ **60.** $\begin{cases} y = x^3 - 2x^2 + x - 1 \\ y = -x^2 + 3x - 1 \end{cases}$

61. $\begin{cases} xy - 1 = 0 \\ 2x - 4y + 7 = 0 \end{cases}$ **62.** $\begin{cases} x - 2y = 1 \\ y = \sqrt{x - 1} \end{cases}$

43.

(0, 1)

45.

(4, 2)

47.

(0, 0), (1, 1)

49.

(0, −13), (±12, 5)

51. (1, 2) **53.** $(-2, 0), \left(\frac{29}{10}, \frac{21}{10}\right)$ **55.** No solution
57. (0.287, 1.75) **59.** (−1, 0), (0, 1), (1, 0)
61. $\left(\frac{1}{2}, 2\right), \left(-4, -\frac{1}{4}\right)$ **63.** 192 units **65.** 3133 units

NAME: Block

SHEET # 323 : LINEAR & NONLINEAR SYSTEM SKILLS.

A. NO CALCULATOR: SOLVE FOR ALL (X,Y) PAIRS. SHOW ALL STEPS.

1. $\begin{cases} x + 2y = 1 \\ 5x - 4y = -23 \end{cases}$ by Substitution.

ANSWERS

1. $(x,y) =$

2. $\begin{cases} x + 7y = 12 \\ 3x - 5y = 10 \end{cases}$ by elimination.

2. $(x,y) =$

3. $\begin{cases} y = x^2 + 2x - 15 \\ y = 0 \end{cases}$ by hand by any method.

That is, solve $x^2 + 2x - 15 = 0$, please.

3. $(x,y) =$

B. CALCULATOR ALLOWED SOLVE FOR ALL (X,Y) PAIRS.

Give accurate answers. Round to 3 decimals.

4. $\begin{cases} y = 2x - 3.6 \\ y = -0.5x + 2.5 \end{cases}$

4. $(x,y) =$

5. $\begin{cases} y = x^2 - 2x + 3 \\ y = -x + 6 \end{cases}$

5. $(x,y) =$

Sheet 324 Lin Quad Sys Review

Name ______________________________ Per/Sec. ________________

For each question (a) Write down each of the "y=" equations, (b) Sketch, (c) Find all intersections (if any).

1. $y = x^2 + 1$
   $x + y = 3$

2. $x - 2y = 1$
   $x = 3y^2 + 1$

3. $x^2 + y^2 = 13$
   $y + 2x = -1$

4. $(x + 1)^2 + (y - 2)^2 = 4$
   $x + y = 0$

5. $x^2 - y^2 = 16$
   $2y = x + 1$

Sheet 324 Lin Quad Sys Review

Name $\boxed{\text{KEY}}$ Per/Sec. ________________

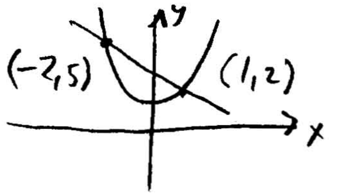
For each question (a) Write down each of the "y=" equations, (b) Sketch, (c) Find all intersections (if any).

1. $y = x^2 + 1$
   $x + y = 3$

$y_1 = x^2 + 1$

$y_2 = 3 - x$

Up-opening parabola



$\boxed{(1,2) \quad (-2,5)}$

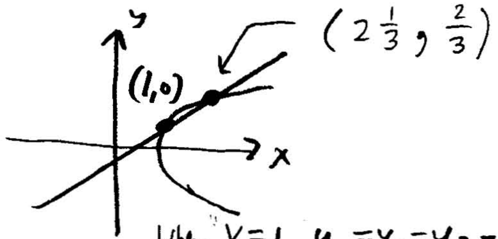
2. $x - 2y = 1$
   $x = 3y^2 + 1$

$y_1 = \frac{1}{2}(x-1)$

$y_2 = \sqrt{\frac{1}{3}(x-1)}$

$y_3 = -\sqrt{\frac{1}{3}(x-1)}$

Right-opening parabola



When $x = 1$, $y_1 = y_2 = y_3 = 0$.

$\boxed{(1,0) \quad (2\frac{1}{3}, \frac{2}{3})}$

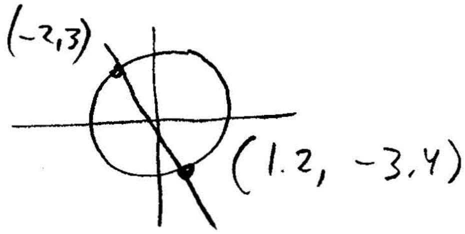
3. $x^2 + y^2 = 13$
   $y + 2x = -1$

$y_1 = -1 - 2x$

$y_2 = \sqrt{13 - x^2}$

$y_3 = -\sqrt{13 - x^2}$

Circle of radius $\sqrt{13}$ Centered at (0,0)



$\boxed{(1.2, -3.4) \quad (-2, 3)}$

4. $(x+1)^2 + (y-2)^2 = 4$
   $x + y = 0$

$(y-2)^2 = 4 - (x+1)^2$

$y - 2 = \pm\sqrt{4 - (x+1)^2}$

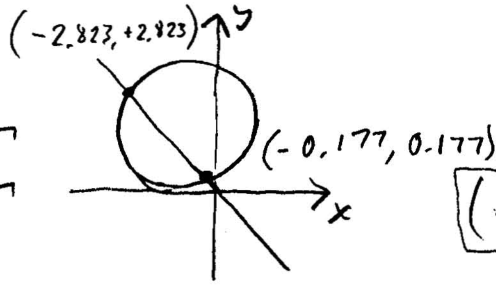
$y = 2 \pm \sqrt{4 - (x+1)^2}$

$y_1 = -x$

$y_2 = 2 + \sqrt{4 - (x+1)^2}$

$y_3 = 2 - \sqrt{4 - (x+1)^2}$

Circle of radius 2 Centered at (-1,2)



$\boxed{(-2.823, 2.823) \quad (-0.177, 0.177)}$

$\boxed{\left(\frac{-3-\sqrt{7}}{2}, \frac{3+\sqrt{7}}{2}\right) \quad \left(\frac{-3+\sqrt{7}}{2}, \frac{3-\sqrt{7}}{2}\right)}$

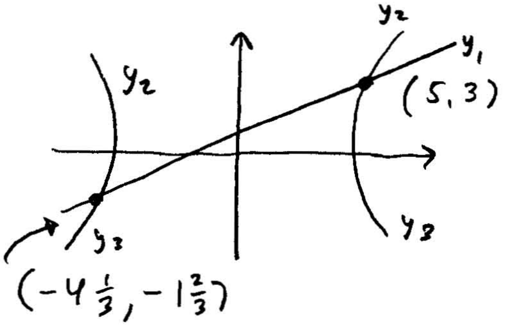
5. $x^2 - y^2 = 16$
   $2y = x + 1$

$y_1 = \frac{1}{2}(x+1)$

$y_2 = \sqrt{x^2 - 16}$

$y_3 = -\sqrt{x^2 - 16}$

Left-Right opening hyperbola.



$\boxed{(5,3) \quad (-4\frac{1}{3}, -1\frac{2}{3})}$

Name: ____________________________

Sheet 325: Intersections with implicit functions

For each equation, (a) Write the name of its shape, (b) Write y_1 and y_2, (c), Graph it on the calculator, (d) Also graph $y_3 = x - 2$, (e) Find the intersection(s), and (f) Sketch the curves and line.

1. $x + y^2 = 3$

   Shape: ______________________

   y_1 =

   y_2 =

   Intersections with y_3

2. $x^2 + y^2 = 9$

   Shape: ______________________

   y_1 =

   y_2 =

   Intersections with y_3

3. $\dfrac{x^2}{9} + y^2 = 1$

   Shape: ______________________

   y_1 =

   y_2 =

   Intersections with y_3

4. $x^2 - y^2 = 9$

   Shape: ______________________

   y_1 =

   y_2 =

   Intersections with y_3

NAME ______________________ DATE __________

Practice with Examples

For use with pages 199–206

GOAL **Add and subtract matrices, multiply a matrix by a scalar, solve matrix equations, and use matrices in real-life situations**

VOCABULARY

A **matrix** is a rectangular arrangement of numbers in rows and columns where the numbers are called **entries.**

The **dimensions** of a matrix are given as *the number of rows* × *the number of columns.*

Scalar multiplication is the process of multiplying each entry in a matrix by a **scalar,** a real number.

EXAMPLE 1 *Adding and Subtracting Matrices*

Perform the indicated operation, if possible.

a. $\begin{bmatrix} 2 \\ -1 \\ 7 \end{bmatrix} + \begin{bmatrix} 4 & 0 & -6 \end{bmatrix}$ **b.** $\begin{bmatrix} 2 & 3 & -5 \\ -1 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 3 \\ 3 & -2 & -1 \end{bmatrix}$

SOLUTION

a. To add or subtract matrices, they must have the same dimensions.

Since $\begin{bmatrix} 2 \\ -1 \\ 7 \end{bmatrix}$ is a 3×1 matrix and $\begin{bmatrix} 4 & 0 & -6 \end{bmatrix}$ is a 1×3 matrix, you cannot add them.

b. Since both matrices are 2×3, you can subtract them.

$$\begin{bmatrix} 2 & 3 & -5 \\ -1 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 3 \\ 3 & -2 & -1 \end{bmatrix} = \begin{bmatrix} 2-0 & 3-1 & -5-3 \\ -1-3 & 0-(-2) & 4-(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & -8 \\ -4 & 2 & 5 \end{bmatrix}$$

Exercises for Example 1

Perform the indicated operation, if possible.

1. $\begin{bmatrix} 2 & 5 \\ -1 & -3 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ 0 & 7 \\ 0 & 0 \end{bmatrix}$ **2.** $\begin{bmatrix} 5 & 0 \\ -2 & 1 \\ 4 & -3 \end{bmatrix} - \begin{bmatrix} 4 & -6 \\ 2 & -2 \\ -1 & 3 \end{bmatrix}$

NAME ______________________________ DATE __________

Practice with Examples

For use with pages 199–206

EXAMPLE 2 Solving a Matrix Equation

Solve the matrix equation for x and y: $-2x\begin{bmatrix} 5 & 0 & -1 \\ 4 & 2 & 8 \end{bmatrix} = \begin{bmatrix} 20 & 0 & -4 \\ 16 & 8 & y \end{bmatrix}$.

SOLUTION

$$-2x\begin{bmatrix} 5 & 0 & -1 \\ 4 & 2 & 8 \end{bmatrix} = \begin{bmatrix} 20 & 0 & -4 \\ 16 & 8 & y \end{bmatrix}$$ Write original equation.

$$\begin{bmatrix} -10x & 0 & 2x \\ -8x & -4x & -16x \end{bmatrix} = \begin{bmatrix} 20 & 0 & -4 \\ 16 & 8 & y \end{bmatrix}$$ Multiply by $-2x$.

$-10x = 20$ $\quad -16x = y$ Equate corresponding entries involving x and y.

$x = -2$ $\quad -16(-2) = y$ Solve the resulting equations.

$32 = y$

Exercises for Example 2

Solve the matrix equation for x and y.

3. $[3x \quad 2 \quad 4] = [9 \quad -y \quad 4]$

4. $\begin{bmatrix} 2y & -1 \\ -6 & 0 \end{bmatrix} + \begin{bmatrix} 5 & 4 \\ x & 8 \end{bmatrix} = \begin{bmatrix} -5 & 3 \\ -7 & 8 \end{bmatrix}$

Chapter 4

LESSON 4.2

NAME ______________________ DATE __________

Practice with Examples

For use with pages 208–213

GOAL Multiply two matrices

VOCABULARY

The product of two matrices A and B is defined only if the number of columns in A is equal to the number of rows in B.

If A is an $m \times n$ matrix and B is an $n \times p$ matrix, then the product AB is an $m \times p$ matrix.

EXAMPLE 1 Finding the Product of Two Matrices

Find AB and BA if $A = \begin{bmatrix} 2 & 1 \\ -2 & 0 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 0 \end{bmatrix}$.

SOLUTION

$$AB = \begin{bmatrix} 2 & 1 \\ -2 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 2 & 0 \end{bmatrix}$$

Because the number of columns in A equals the number of rows in B, the product is defined. AB will be a 3×2 matrix.

$$= \begin{bmatrix} 2(4) + 1(2) & 2(3) + 1(0) \\ -2(4) + 0(2) & -2(3) + 0(0) \\ 0(4) + 3(2) & 0(3) + 3(0) \end{bmatrix}$$

Multiply corresponding entries in the first row of A and the first column of B. Then add. Use a similar procedure to write the other entries.

$$= \begin{bmatrix} 10 & 6 \\ -8 & -6 \\ 6 & 0 \end{bmatrix}$$

BA is undefined because B is a 2×2 matrix and A is a 3×2 matrix. The number of columns in B does not equal the number of rows in A.

Exercises for Example 1

Find the product. If it is not defined, state the reason.

1. $\begin{bmatrix} 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} -1 & 4 \\ 0 & 1 \\ 5 & 2 \end{bmatrix}$

2. $\begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} -1 & 5 \\ 6 & 2 \end{bmatrix}$

Chapter 4

NAME ______________________ DATE __________

Practice with Examples

For use with pages 208–213

3. $\begin{bmatrix} 2 & -1 & 0 \\ 0 & 5 & 3 \\ 1 & -2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 2 & 3 & 3 \end{bmatrix}$

4. $\begin{bmatrix} 0 & 1 \\ 4 & 3 \\ 5 & -1 \end{bmatrix}\begin{bmatrix} 2 & -1 & 0 \\ 3 & 4 & 1 \end{bmatrix}$

EXAMPLE 2 Using Matrix Operations

If $A = \begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 5 \\ 2 & 1 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$, simplify each expression.

a. $A(BC)$

b. $(AB)C$

SOLUTION

a. $A(BC) = \begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix}\left(\begin{bmatrix} -3 & 5 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}\right)$ Substitute the matrices for A, B, and C.

$= \begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -3 & 1 \\ 2 & -5 \end{bmatrix}$ Multiply B by C first.

$= \begin{bmatrix} -6 & -11 \\ 3 & -1 \end{bmatrix}$ Multiply A by the result.

b. $(AB)C = \left(\begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -3 & 5 \\ 2 & 1 \end{bmatrix}\right)\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$ Substitute the matrices for A, B, and C.

$= \begin{bmatrix} -6 & 23 \\ 3 & -5 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$ Multiply A by B first.

$= \begin{bmatrix} -6 & -11 \\ 3 & -1 \end{bmatrix}$ Multiply the result by C.

LESSON 4.2 CONTINUED

NAME ______________________ DATE __________

Practice with Examples

For use with pages 208–213

Exercises for Example 2

Use the given matrices to simplify the expression.

$$A = \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix},\ B = \begin{bmatrix} 4 & -3 \\ 0 & 2 \end{bmatrix},\ C = \begin{bmatrix} -2 & -3 \\ 1 & 1 \end{bmatrix}$$

5. AA

6. $A(B + C)$

7. $2(CA)$

8. $AB + BC$

9. $A(B - C)$

10. $-2(BA)$

Chapter 4

NAME ______________________ DATE __________

Practice with Examples

For use with pages 214–221

GOAL **Evaluate determinants of 2 × 2 and 3 × 3 matrices**

VOCABULARY

The **determinant** of a square matrix A is denoted by det A or $|A|$.

Cramer's Rule is a method of solving a system of linear equations using the determinant of the coefficient matrix of the linear system.

The entries in the **coefficient matrix** are the coefficients of the variables in the same order.

The area of a triangle with vertices (x_1, y_2), (x_2, y_2), and (x_3, y_3) is given by

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

where the symbol ± indicates that the appropriate sign should be chosen to yield a positive value.

Determinant of a 2 × 2 Matrix

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

The determinant of a 2 × 2 matrix is the difference of the products of the entries on the diagonals.

Determinant of a 3 × 3 Matrix

1. Repeat the first two columns to the right of the determinant.
2. Subtract the sum of the products of the entries on the diagonals going *up* from left to right from the sum of the products of the entries on the diagonals going *down* from left to right.

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \left|\begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix}\right|\begin{matrix} a & b \\ d & e \\ g & h \end{matrix} = (aei + bfg + cdh) - (gec + hfa + idb)$$

Chapter 4

NAME ______________________________ DATE __________

Practice with Examples

For use with pages 214–221

EXAMPLE 1 The Area of a Triangle

Find the area of the triangle with the vertices $A(-3, -1)$, $B(2, 0)$, $C(2, -4)$.

SOLUTION

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} -3 & -1 & 1 \\ 2 & 0 & 1 \\ 2 & -4 & 1 \end{vmatrix}\begin{matrix} -3 & -1 \\ 2 & 0 \\ 2 & -4 \end{matrix}$$

Write the determinant, repeating the first two columns at the end.

$$= \pm\tfrac{1}{2}([0 + (-2) + (-8)] - [0 + 12 + (-2)])$$

Find the products of the diagonals.

$$= \pm\tfrac{1}{2}[-10 - 10]$$

Simplify.

$$= 10$$

Multiply by $-\frac{1}{2}$ to get a positive value.

Exercises for Example 1

Find the area of the triangle with the given vertices.

1. $A(2, 3)$, $B(0, 5)$, $C(-1, -2)$

2. $A(0, 4)$, $B(3, 5)$, $C(-1, 4)$

3. $A(-1, -2)$, $B(2, 1)$, $C(0, 3)$

4. $A(1, 2)$, $B(2, 6)$, $C(3, 2)$

Chapter 4

NAME ______________________ DATE ____________

Practice with Examples

For use with pages 214–221

EXAMPLE 2 Using Cramer's Rule for a 2 × 2 System

Use Cramer's Rule to solve this system: $3x - 2y = 22$
$x + 4y = -2$

SOLUTION

$\begin{vmatrix} 3 & -2 \\ 1 & 4 \end{vmatrix} = 12 - (-2) = 14$ Evaluate the determinant of the coefficient matrix.

$x = \dfrac{\begin{vmatrix} 22 & -2 \\ -2 & 4 \end{vmatrix}}{14} = \dfrac{88 - 4}{14} = 6$ Since the determinant is not 0, apply Cramer's Rule.

$y = \dfrac{\begin{vmatrix} 3 & 22 \\ 1 & -2 \end{vmatrix}}{14} = \dfrac{-6 - 22}{14} = -2$

The solution is $(6, -2)$.

✓ Check this solution in the original equations.

$$3x - 2y = 22 \qquad x + 4y = -2$$
$$3(\mathbf{6}) - 2(-\mathbf{2}) \stackrel{?}{=} 22 \qquad \mathbf{6} + 4(-\mathbf{2}) \stackrel{?}{=} -2$$
$$22 = 22 \checkmark \qquad -2 = -2 \checkmark$$

Exercises for Example 2

Use Cramer's Rule to solve the linear system.

5. $2x + y = 1$
$-x + y = 7$

6. $3x + 4y = 2$
$2x + y = 3$

7. $x + y = 5$
$2x - y = 4$

8. $6x - 3y = 39$
$5x + 9y = -25$

9. $3x - 2y = 8$
$4x - 3y = 10$

10. $5x - 2y = -9$
$-7x + 3y = 14$

LESSON 4.4

NAME ____________________ DATE __________

Practice with Examples

For use with pages 223–229

GOAL **Find and use inverse matrices and use inverse matrices in real-life situations**

VOCABULARY

The $n \times n$ **identity matrix** is the matrix that has 1's on the main diagonal and 0's elsewhere.

Two $n \times n$ matrices are **inverses** of each other if their product (in *both* orders) is the $n \times n$ identity matrix.

The inverse of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \text{ provided } ad - cb \neq 0.$$

EXAMPLE 1 *Finding the Inverse of 2 × 2 Matrix*

Find the inverse of $A = \begin{bmatrix} -3 & -7 \\ 2 & 5 \end{bmatrix}$.

SOLUTION

$|A| = (-3)(5) - (2)(-7) = -1$ Evaluate the determinant of A.

$A^{-1} = \frac{1}{-1}\begin{bmatrix} 5 & 7 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} -5 & -7 \\ 2 & 3 \end{bmatrix}$ Use the formula $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Exercises for Example 1

Find the inverse of the matrix.

1. $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$ **2.** $\begin{bmatrix} 2 & 5 \\ 4 & 11 \end{bmatrix}$ **3.** $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

Chapter 4

NAME ______________________ DATE ________

Practice with Examples

For use with pages 223–229

EXAMPLE 2 Solving a Matrix Equation

Solve the matrix equation $\underbrace{\begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}}_{A} X = \underbrace{\begin{bmatrix} 12 & 2 \\ 17 & 3 \end{bmatrix}}_{B}$ for the 2×2 matrix X.

SOLUTION

$A^{-1} = \frac{1}{9-8}\begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}$ Find the inverse of A, A^{-1}.

To solve for X, multiply both sides of the equation by A^{-1} on the *left*.

$\begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}X = \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} 12 & 2 \\ 17 & 3 \end{bmatrix}$ $A^{-1}AX = A^{-1}B$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}X = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}$$

$\begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ 17 & 3 \end{bmatrix} = B$ Check by multiplying A and X to see if you get B.

Exercises for Example 2

Solve the matrix equation.

4. $\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}X = \begin{bmatrix} 12 & 2 \\ 16 & -5 \end{bmatrix}$

5. $\begin{bmatrix} 4 & -6 \\ 3 & -4 \end{bmatrix}X = \begin{bmatrix} 6 & 0 \\ 5 & 1 \end{bmatrix}$

Chapter 4

NAME ______________________ DATE ____________

Practice with Examples

For use with pages 223–229

EXAMPLE 3 Encoding a Message

Use $A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$ to encode the message GO NORTH.

Solution

First, convert the letters to row matrices, inserting a 0 for spaces between words.

$[7 \quad 15][0 \quad 14][15 \quad 18][20 \quad 8]$

Then multiply each of the uncoded row matrices by matrix A on the *right* to obtain coded row matrices.

__ = 0	F = 6	L = 12	R = 18	X = 24
A = 1	G = 7	M = 13	S = 19	Y = 25
B = 2	H = 8	N = 14	T = 20	Z = 26
C = 3	I = 9	O = 15	U = 21	
D = 4	J = 10	P = 16	V = 22	
E = 5	K = 11	Q = 17	W = 23	

$[7 \quad 15]\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix} = [6 \quad 30]$ $\qquad [15 \quad 18]\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix} = [27 \quad 36]$

$[0 \quad 14]\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix} = [-14 \quad 28]$ $\qquad [20 \quad 8]\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix} = [52 \quad 16]$

The coded message is 6, 30, __, 14, 28, 27, 36, 52, 16.

Exercises for Example 3

Use the code above and the matrix to encode the message.

6. HELLO

$A = \begin{bmatrix} -2 & 1 \\ 3 & -3 \end{bmatrix}$

7. CALL ME

$A = \begin{bmatrix} 4 & -3 \\ 2 & 0 \end{bmatrix}$

8. I WON

$A = \begin{bmatrix} -1 & 5 \\ 3 & -2 \end{bmatrix}$

LESSON 4.5

NAME ______________________ DATE __________

Practice with Examples

For use with pages 230–236

GOAL **Solve systems of linear equations using inverse matrices and use systems of linear equations to solve real-life problems**

VOCABULARY

For a linear system of equations written as a matrix equation $AX = B$, the matrix A is the coefficient matrix of the system, X is the **matrix of variables,** and B is the **matrix of constants.** If the determinant of A is nonzero, then the linear system has exactly one solution, which is $X = A^{-1}B$.

EXAMPLE 1 *Solving a Linear System*

Use matrices to solve the linear system. $5x + 2y = 3$ **Equation 1**

$4x + 2y = 4$ **Equation 2**

SOLUTION

$$\begin{array}{ccc} A & X & B \\ \begin{bmatrix} 5 & 2 \\ 4 & 2 \end{bmatrix} & \begin{bmatrix} x \\ y \end{bmatrix} = & \begin{bmatrix} 3 \\ 4 \end{bmatrix} \end{array}$$

Write the matrix equation for the system.

$$A^{-1} = \frac{1}{10 - 8}\begin{bmatrix} 2 & -2 \\ -4 & 5 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & \frac{5}{2} \end{bmatrix}$$

Find the inverse of matrix A.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & \frac{5}{2} \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

To find X, multiply B by A^{-1} on the **left.**

The solution is $(-1, 4)$.

Exercises for Example 1

Use an inverse matrix to solve the linear system.

1. $2x - y = 1$

$-3x + 2y = 0$

2. $3x + 4y = -4$

$4x + 5y = -7$

3. $6x - 5y = 3$

$3x - 2y = 3$

Chapter 4

NAME ______________________________ DATE ____________

Practice with Examples

For use with pages 230–236

EXAMPLE 2 Writing and Using a Linear System

A chemist wants to use three different solutions to create a 50-liter mixture containing 32% acid. The first solution contains 10% acid, the second 30%, and the third 50%. He needs to use twice as much of the 50% solution as the 30% solution. How many liters of each solution should be used?

SOLUTION

Verbal Model

$$\boxed{\text{Amount of 10\%}} + \boxed{\text{Amount of 30\%}} + \boxed{\text{Amount of 50\%}} = \boxed{\text{Total amount}}$$

$$0.10\boxed{\text{Amount of 10\%}} + 0.30\boxed{\text{Amount of 30\%}} + 0.50\boxed{\text{Amount of 50\%}} = 0.32\boxed{\text{Total amount}}$$

$$\boxed{\text{Amount of 50\%}} = 2\boxed{\text{Amount of 30\%}}$$

Labels

Amount of 10% $= f$

Amount of 30% $= s$

Amount of 50% $= t$

Total amount $= 50$

Algebraic Model

$f + s + t = 50$ Equation 1

$0.10f + 0.30s + 0.50t = 0.32(50)$ Equation 2

$t = 2s$ Equation 3

$f + s + t = 50$

$0.10f + 0.30s + 0.50t = 16$

$-2s + t = 0$

$$\begin{array}{ccc} A & X & B \\ \begin{bmatrix} 1 & 1 & 1 \\ 0.1 & 0.3 & 0.5 \\ 0 & -2 & 1 \end{bmatrix} & \begin{bmatrix} f \\ s \\ t \end{bmatrix} = & \begin{bmatrix} 50 \\ 16 \\ 0 \end{bmatrix} \end{array}$$

Using a graphing calculator, enter the coefficient matrix A and the matrix of constants B. To find X, multiply A^{-1} by B.

```
Matrix [A]        3 x 3
[ 1      1        1]
[.1     .3       .5]
[ 0     -2        1]
```

```
Matrix [B]        3 x 1
[50     ]
[16     ]
[ 0     ]
```

```
[A]⁻¹[B]
[[17    ]
 [11    ]
 [22    ]]
```

The chemist should use 17 liters of 10% solution, 11 liters of 30% solution and 22 liters of 50% solution.

Chapter 4

NAME ______________________________ DATE __________

Practice with Examples

For use with pages 230–236

Exercises for Example 2

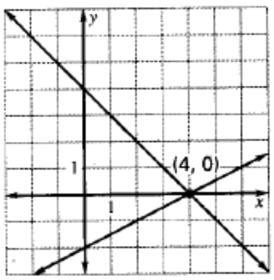
4. You can purchase peanuts for $3 per pound, almonds for $4 per pound, and cashews for $8 per pound. You want to create 140 pounds of a mixture that costs $6 per pound. If twice as many peanuts are used than almonds, how many pounds of each type should be used?

5. Suppose the chemist in Example 2 wants to create a 600-liter mixture containing 25% acid. This time, the first solution contains 30% acid, the second 20%, and the third 15%. If the mixture is to contain 100 more liters of the 15% solution than the 20% solution, how many liters of each solution should be used?

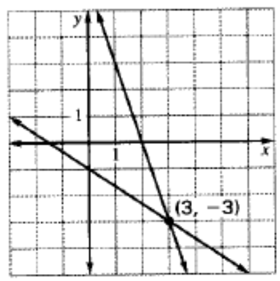
Answers

Lesson 3.1

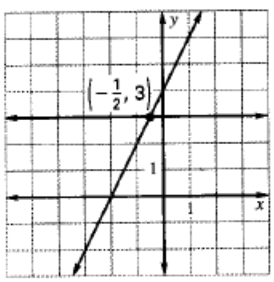
1. $(4, 0)$



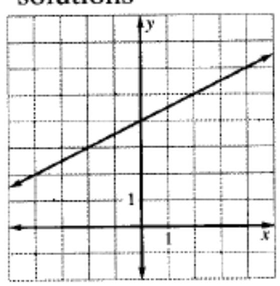
2. $(3, -3)$



3. $\left(-\frac{1}{2}, 3\right)$



4. infinitely many solutions



Lesson 3.2

1. $\left(\frac{1}{2}, -5\right)$ **2.** $(-1, 3)$ **3.** $(2, 4)$ **4.** $(2, -1)$
5. $(-3, -2)$ **6.** $\left(\frac{1}{2}, 4\right)$ **7.** $\left(\frac{2}{3}, -3\right)$
8. no solution **9.** $(-3, 5)$ **10.** $\left(\frac{1}{2}, 1\right)$
11. $(5, -9)$ **12.** infinitely many solutions

Lesson 4.1

1. not possible **2.** $\begin{bmatrix} 1 & 6 \\ -4 & 3 \\ 5 & -6 \end{bmatrix}$

3. $x = 3;\ y = -2$ **4.** $x = -1;\ y = -5$

5. $\begin{bmatrix} \$670.23 & \$1665.49 \\ \$453.84 & \$1193.10 \\ \$489.28 & \$1247.62 \end{bmatrix}$

6. $\begin{bmatrix} \$704.43 & \$1750.46 \\ \$476.99 & \$1253.97 \\ \$514.25 & \$1311.27 \end{bmatrix}$

Lesson 4.2

1. $\begin{bmatrix} 18 & 19 \end{bmatrix}$ **2.** $\begin{bmatrix} 4 & 12 \\ -15 & 11 \end{bmatrix}$

3. undefined; the number of columns of the first matrix does not equal the number of rows of the second matrix.

4. $\begin{bmatrix} 3 & 4 & 1 \\ 17 & 8 & 3 \\ 7 & -9 & -1 \end{bmatrix}$ **5.** $\begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}$

6. $\begin{bmatrix} -1 & 9 \\ 3 & -15 \end{bmatrix}$ **7.** $\begin{bmatrix} -8 & 2 \\ 2 & 0 \end{bmatrix}$

8. $\begin{bmatrix} -15 & -10 \\ 10 & -6 \end{bmatrix}$ **9.** $\begin{bmatrix} -7 & 1 \\ 13 & -1 \end{bmatrix}$ **10.** $\begin{bmatrix} 20 & -14 \\ -8 & 4 \end{bmatrix}$

Lesson 4.3

1. 8 **2.** $\frac{1}{2}$ **3.** 6 **4.** 4 **5.** $(-2, 5)$
6. $(2, -1)$ **7.** $(3, 2)$ **8.** $(4, -5)$ **9.** $(4, 2)$
10. $(1, 7)$

Lesson 4.4

1. $\begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}$ **2.** $\begin{bmatrix} \frac{11}{2} & -\frac{5}{2} \\ -2 & 1 \end{bmatrix}$

3. $\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$ **4.** $\begin{bmatrix} 1 & 4 \\ 5 & -3 \end{bmatrix}$ **5.** $\begin{bmatrix} 3 & 3 \\ 1 & 2 \end{bmatrix}$

6. $-1, -7, 12, -24, -30, 15$
7. $14, -9, 72, -36, 26, 0, 20, -15$
8. $-9, 45, 22, 85, -14, 70$

Lesson 4.5

1. $(2, 3)$ **2.** $(-8, 5)$ **3.** $(3, 3)$ **4.** 40 lbs of peanuts, 20 lbs of almonds, 80 lbs of cashews
5. 380 L of 30%, 60 L of 20%, 160 L of 15%

Using TI-83 or TI-84 for Matrix algebra

<table>
<tr><th colspan="4">Entering and Editing a Matrix</th></tr>
<tr>
<td>Press MATRIX, right arrow to EDIT.
Press Number 1 or highlight 1 and press ENTER.</td>
<td>NAMES MATH EDIT
1:[A] 3×3
2:[B] 2×2
3:[C] 2×2
4:[D] 3×3
5:[E] 2×2
6:[F] 2×1
7↓[G] 2×7</td>
<td>Matrix A is a 3x3. Press ENTER after each number.
To save the matrix, press QUIT [2nd MODE].</td>
<td>MATRIX[A] 3 ×3
[2 -17 11]
[-1 11 -7]
[0 3 -2]
3,3=-2</td>
</tr>
<tr><th colspan="4">Finding the Inverse and the Determinant of Matrix A</th></tr>
<tr>
<td>To check the entries in Matrix A, press MATRIX, number 1, and ENTER.</td>
<td>[A]
[[2 -17 11]
[-1 11 -7]
[0 3 -2]]</td>
<td>To find
A inverse, press MATRIX, number 1,
x^{-1}, and ENTER.</td>
<td>[A]-1
[[1 1 2]
[2 4 -3]
[3 6 -5]]</td>
</tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr>
<td>To find the product of A inverse and A, press MATRIX, number 1, x^{-1}, MATRIX, number 1, and ENTER.</td>
<td>[A]-1[A]
[[1 0 0]
[0 1 0]
[0 0 1]]</td>
<td>To find det(A), press MATRIX and the right arrow to highlight MATH.</td>
<td>NAMES MATH EDIT
1:det(
2:T
3:dim(
4:Fill(
5:identity(
6:randM(
7↓augment(</td>
</tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr>
<td>Press number 1, MATRIX, number 1, close), and ENTER.
det([A]) = -1</td>
<td>det([A])
-1</td>
<td></td>
<td></td>
</tr>
</table>

Solve using Calculator (inverse matrix method)

$$\begin{aligned} x - y + z &= 3 \\ 2x + y - 3z &= 5 \\ 4x + y - z &= 11 \end{aligned}$$

<table>
<tr><th colspan="4">Solve a System of Equations Using Matrices: $X = A^{-1}B$</th></tr>
<tr>
<td>Enter the coefficients of the variables as Matrix A and the constants as Matrix B as shown on the right.</td>
<td>[[1 -1 1]
[2 1 -3]
[4 1 -1]]
[B]
[[3]
[5]
[11]]</td>
<td>Press Matrix, number 1, and x^{-1} to get $[A]^{-1}$.

Press Matrix, number 2 to get [B].

Once $[A]^{-1}[B]$ is on screen, press Enter to get the answer.

So our solution is (2.8, 0, 0.2).</td>
<td>[A]-1[B]
[[2.8]
[0]
[.2]]</td>
</tr>
</table>

2-4 NAME ______________________ DATE ______________

Practice Worksheet

Solving Systems of Equations by Using Matrices

Solve each system of equations

1. $4x-3y=1$
$3x+2y=5$

2. $3x-3y=9$
$12x-8y=40$

3. $x+y+z=3$
$3x-2y-z=-4$
$x+y-z=-1$

4. $x+y=5$
$3x+z=2$
$4y-z=8$

5. $4x-3y-6z=2$
$2x+3y+3z=2$
$10x-6y+3z=0$

6. $3x-5y+z=8$
$7x+y=4$
$4y-z=10$

7. $x-y+z=1$
$x-2y+z=2$
$2x-y+z=-1$

8. $2x-3y-4z=2$
$-6x-6y+6z=5$
$4x+4y-2z=3$

9. $4x-6y+2z=-9$
$2x+4y-2z=9$
$x-y+3z=-4$

10. $x-z=5$
$2x+y=7$
$y+3z=12$

2-4 NAME ______________________ DATE ____________

Practice Worksheet

Solving Systems of Equations by Using Matrices

Solve each system of equations

1. $4x - 3y = 1$
$3x + 2y = 5$

(1, 1) ✓

2. $3x - \overset{3}{2}y = 9$
$12x - 8y = 40$

(4, 1) ✓

3. $x + y + z = 3$
$3x - 2y - z = -4$
$x + y - z = -1$

(0, 1, 2) ✓

4. $x + y = 5$
$3x + z = 2$
$4y - z = 8$

(10, −5, −28) ✓

5. $4x - 3y - 6z = 2$
$2x + 3y + 3z = 2$
$10x - 6y + 3z = 0$

$\left(\frac{1}{2}, \frac{2}{3}, -\frac{1}{3}\right)$ ✓

6. $3x - 5y + z = 8$
$7x + y = 4$
$4y - z = 10$

(2.2, −11.4, −55.6) ✓

7. $x - y + z = 1$
$x - 2y + z = 2$
$2x - y + z = -1$

(−2, −1, 2) ✓

8. $2x - 3y - 4z = 2$
$-6x - 6y + 6z = 5$
$4x + 4y - 2z = 3$

$\left(\frac{13}{3}, -2, \frac{19}{6}\right)$ ✓

9. $4x - 6y + 2z = -9$
$2x + 4y - 2z = 9$
$x - y + 3z = -4$

$\left(\frac{1}{2}, \frac{3}{2}, -1\right)$ ✓

10. $x - z = 5$
$2x + y = 7$
$y + 3z = 12$

(20, −33, 15) ✓

Name________________________________

Matrix Multiplication

Date________________ Period____

Simplify. Write "undefined" for expressions that are undefined.

1) $\begin{bmatrix} 0 & 2 \\ -2 & -5 \end{bmatrix} \cdot \begin{bmatrix} 6 & -6 \\ 3 & 0 \end{bmatrix}$

2) $\begin{bmatrix} 6 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} -5 & 4 \end{bmatrix}$

3) $\begin{bmatrix} -5 & -5 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} -2 & -3 \\ 3 & 5 \end{bmatrix}$

4) $\begin{bmatrix} -3 & 5 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 & -2 \\ 1 & -5 \end{bmatrix}$

5) $\begin{bmatrix} 0 & 5 \\ -3 & 1 \\ -5 & 1 \end{bmatrix} \cdot \begin{bmatrix} -4 & 4 \\ -2 & -4 \end{bmatrix}$

6) $\begin{bmatrix} 5 & 3 & 5 \\ 1 & 5 & 0 \end{bmatrix} \cdot \begin{bmatrix} -4 & 2 \\ -3 & 4 \\ 3 & -5 \end{bmatrix}$

7) $\begin{bmatrix} -5 \\ 6 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \end{bmatrix}$

8) $\begin{bmatrix} 3 & 2 & 5 \\ 2 & 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 & 5 & -5 \\ 5 & -1 & 6 \end{bmatrix}$

9) $\begin{bmatrix} 3 & -1 \\ -3 & 6 \\ -6 & -6 \end{bmatrix} \cdot \begin{bmatrix} -1 & 6 \\ 5 & 4 \end{bmatrix}$

10) $\begin{bmatrix} 5 & 4 \\ 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ 3 \end{bmatrix}$

11) $\begin{bmatrix} -1 & 1 & -1 \\ 5 & 2 & -5 \\ 6 & -5 & 1 \\ -5 & 6 & 0 \end{bmatrix} \cdot \begin{bmatrix} 6 & 5 \\ 5 & -6 \\ 6 & 0 \end{bmatrix}$

12) $\begin{bmatrix} -2 & -6 \\ -4 & 3 \\ 5 & 0 \\ 4 & -6 \end{bmatrix} \cdot \begin{bmatrix} 2 & -2 & 2 \\ -2 & 0 & -3 \end{bmatrix}$

13) $\begin{bmatrix} 2 & -5v \end{bmatrix} \cdot \begin{bmatrix} -5u & -v \\ 0 & 6 \end{bmatrix}$

14) $\begin{bmatrix} -4 & -y \\ -2x & -4 \end{bmatrix} \cdot \begin{bmatrix} -4x & 0 \\ 2y & -5 \end{bmatrix}$

Critical thinking questions:

15) Write an example of a matrix multiplication that is undefined.

16) In the expression $A \cdot B$, if A is a 3×5 matrix then what could be the dimensions of B?

Kuta Software - Infinite Algebra 2

Name________________________________

Matrix Multiplication

Date________________ Period____

Simplify. Write "undefined" for expressions that are undefined.

1) $\begin{bmatrix} 0 & 2 \\ -2 & -5 \end{bmatrix} \cdot \begin{bmatrix} 6 & -6 \\ 3 & 0 \end{bmatrix}$

$\begin{bmatrix} 6 & 0 \\ -27 & 12 \end{bmatrix}$

2) $\begin{bmatrix} 6 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} -5 & 4 \end{bmatrix}$

$\begin{bmatrix} -30 & 24 \\ 15 & -12 \end{bmatrix}$

3) $\begin{bmatrix} -5 & -5 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} -2 & -3 \\ 3 & 5 \end{bmatrix}$

$\begin{bmatrix} -5 & -10 \\ 8 & 13 \end{bmatrix}$

4) $\begin{bmatrix} -3 & 5 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 & -2 \\ 1 & -5 \end{bmatrix}$

$\begin{bmatrix} -13 & -19 \\ -11 & -1 \end{bmatrix}$

5) $\begin{bmatrix} 0 & 5 \\ -3 & 1 \\ -5 & 1 \end{bmatrix} \cdot \begin{bmatrix} -4 & 4 \\ -2 & -4 \end{bmatrix}$

$\begin{bmatrix} -10 & -20 \\ 10 & -16 \\ 18 & -24 \end{bmatrix}$

6) $\begin{bmatrix} 5 & 3 & 5 \\ 1 & 5 & 0 \end{bmatrix} \cdot \begin{bmatrix} -4 & 2 \\ -3 & 4 \\ 3 & -5 \end{bmatrix}$

$\begin{bmatrix} -14 & -3 \\ -19 & 22 \end{bmatrix}$

7) $\begin{bmatrix} -5 \\ 6 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \end{bmatrix}$

$\begin{bmatrix} -15 & 5 \\ 18 & -6 \\ 0 & 0 \end{bmatrix}$

8) $\begin{bmatrix} 3 & 2 & 5 \\ 2 & 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 & 5 & -5 \\ 5 & -1 & 6 \end{bmatrix}$

Undefined

9) $\begin{bmatrix} 3 & -1 \\ -3 & 6 \\ -6 & -6 \end{bmatrix} \cdot \begin{bmatrix} -1 & 6 \\ 5 & 4 \end{bmatrix}$

$\begin{bmatrix} -8 & 14 \\ 33 & 6 \\ -24 & -60 \end{bmatrix}$

10) $\begin{bmatrix} 5 & 4 \\ 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ 3 \end{bmatrix}$

$\begin{bmatrix} -8 \\ -11 \end{bmatrix}$

11) $\begin{bmatrix} -1 & 1 & -1 \\ 5 & 2 & -5 \\ 6 & -5 & 1 \\ -5 & 6 & 0 \end{bmatrix} \cdot \begin{bmatrix} 6 & 5 \\ 5 & -6 \\ 6 & 0 \end{bmatrix}$

$\begin{bmatrix} -7 & -11 \\ 10 & 13 \\ 17 & 60 \\ 0 & -61 \end{bmatrix}$

12) $\begin{bmatrix} -2 & -6 \\ -4 & 3 \\ 5 & 0 \\ 4 & -6 \end{bmatrix} \cdot \begin{bmatrix} 2 & -2 & 2 \\ -2 & 0 & -3 \end{bmatrix}$

$\begin{bmatrix} 8 & 4 & 14 \\ -14 & 8 & -17 \\ 10 & -10 & 10 \\ 20 & -8 & 26 \end{bmatrix}$

13) $\begin{bmatrix} 2 & -5v \end{bmatrix} \cdot \begin{bmatrix} -5u & -v \\ 0 & 6 \end{bmatrix}$

$\begin{bmatrix} -10u & -32v \end{bmatrix}$

14) $\begin{bmatrix} -4 & -y \\ -2x & -4 \end{bmatrix} \cdot \begin{bmatrix} -4x & 0 \\ 2y & -5 \end{bmatrix}$

$\begin{bmatrix} 16x - 2y^2 & 5y \\ 8x^2 - 8y & 20 \end{bmatrix}$

Critical thinking questions:

15) Write an example of a matrix multiplication that is undefined.

Many answers. Ex: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

16) In the expression $A \cdot B$, if A is a 3×5 matrix then what could be the dimensions of B?

$5 \times$ Anything

Multiplicative Properties: Commutative, Identity, Inverse

$$A = \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix} \quad C = \begin{bmatrix} -2 & -3 \\ 1 & 1 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad R = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$$

$AC =$

$CA =$

$AI =$

$IA =$

$RA =$

$AR =$

Sheet #431

Determinants of 2x2, 3x3 and 4x4 square matrices

$$A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad \begin{pmatrix} + & - \\ - & + \end{pmatrix}$$

$$A=\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$$

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \quad \begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$$

$$A=\begin{pmatrix} 0 & 2 & 1 \\ 3 & -1 & 2 \\ 4 & 0 & 1 \end{pmatrix}$$

$$A=\begin{pmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ -3 & -2 & 0 \end{pmatrix}$$

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$$

$$A=\begin{pmatrix} 1 & -2 & 3 & 0 \\ -1 & 1 & 0 & 2 \\ 0 & 2 & 0 & 3 \\ 3 & 4 & 0 & 2 \end{pmatrix}$$

Inverse of a square matrix

Written by Paul Bourke
August 2002

See also: Determinant of a Square Matrix

The inverse of a square matrix A with a non zero determinant is the adjoint matrix divided by the determinant, this can be written as

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A)$$

The adjoint matrix is the transpose of the cofactor matrix. The cofactor matrix is the matrix of determinants of the minors A_{ij} multiplied by -1^{i+j}. The i,j'th minor of A is the matrix A without the i'th column or the j'th row.

Example (3x3 matrix)

The following example illustrates each matrix type and at 3x3 the steps can be readily calculated on paper. It can also be verified that the original matrix A multipled by its inverse gives the identity matrix (all zeros except along the diagonal which are ones).

$$A = \begin{vmatrix} 1 & 2 & 0 \\ -1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

Determinant = 9

$$\text{Cofactor matrix} = \begin{vmatrix} +\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} & -\begin{vmatrix} -1 & 1 \\ 1 & 3 \end{vmatrix} & +\begin{vmatrix} -1 & 1 \\ 1 & 2 \end{vmatrix} \\ -\begin{vmatrix} 2 & 0 \\ 2 & 3 \end{vmatrix} & +\begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} \\ +\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} \end{vmatrix} = \begin{vmatrix} 1 & 4 & -3 \\ -6 & 3 & -0 \\ 2 & -1 & 3 \end{vmatrix}$$

$$\text{Adjoint matrix} = \text{Transpose of cofactor matrix} = \begin{vmatrix} 1 & -6 & 2 \\ 4 & 3 & -1 \\ -3 & -0 & 3 \end{vmatrix}$$

$$A^{-1} = \text{Inverse of } A = \begin{vmatrix} 1/9 & -6/9 & 2/9 \\ 4/9 & 3/9 & -1/9 \\ -3/9 & 0 & 3/9 \end{vmatrix}$$

$$A\,A^{-1} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Inverse of a 2x2 matrix

The inverse of a 2x2 matrix can be written explicitly, namely

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \frac{1}{ad - bc} \begin{vmatrix} d & -b \\ -c & a \end{vmatrix}$$

Source code

Sheet #451

Name: Block:

Matrix equations and systems of equations.

Part A: Graphing/matrix calculator allowed.

1. Solve. Check your answers.

$$\begin{cases} -x + 2y - 3z = -6 \\ 2x - y - z = -3 \\ x + 3y - 5z = -9 \end{cases}$$

(a) Write as an augmented matrix.

(b) Solve using the reduced row echelon form (rref).

2. You can purchase peanuts for \$2.50 per pound, almonds for \$6.00 per pound, and cashews for \$8.00 per pound. You want to make a 1,000-pound mixture that costs \$5.00 per pound.

(a) You want 3 times as many peanuts as cashews. How many pounds of each type of nut should be used?

Write the equations.

Give answers using both words and numbers, with numbers rounded to the nearest tenth of a pound.

Part B: No graphing calculator. Regular calculator OK.

3. Consider the matrix

$$A = \begin{bmatrix} 1 & -1 \\ -2 & 6 \end{bmatrix}$$

(a) Write the determinant. Find it.

(b) Showing the math, find the inverse.

(c) Use the inverse of the matrix A to solve for the matrix X.

$$\begin{bmatrix} 1 & -1 \\ -2 & 6 \end{bmatrix} X = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$$

Write the equation for finding X.

Find X.

(d) Solve using the inverse matrix method.

$$\begin{cases} x - y = 5 \\ -2x + 6y = -24 \end{cases}$$

Rewrite the system of equations as a matrix equation.

Write the matrix equation for finding x and y.

Find x and y.

Sheet #451 Name: $\boxed{\text{KEY}}$ Block:

Matrix equations and systems of equations.

Part A: Graphing/matrix calculator allowed.

1. Solve. Check your answers.

$$\begin{cases} -x + 2y - 3z = -6 \\ 2x - y - z = -3 \\ x + 3y - 5z = -9 \end{cases}$$

(a) Write as an augmented matrix.

$$\left[\begin{array}{ccc|c} -1 & 2 & -3 & -6 \\ 2 & -1 & -1 & -3 \\ 1 & 3 & -5 & -9 \end{array}\right]$$

(b) Solve using the reduced row echelon form (rref).

$\boxed{x = 6/11 \quad y = 15/11 \quad z = 30/11}$

2. You can purchase peanuts for \$2.50 per pound, almonds for \$6.00 per pound, and cashews for \$8.00 per pound. You want to make a 1,000-pound mixture that costs \$5.00 per pound.

Let
p = # pounds of peanuts
a = # pounds of almonds
c = # pounds of cashews.

(a) If a ratio (r) of 3 pounds of peanuts to one pound of cashews is used, how many pounds of each type should be used? Answer to the nearest pound. $p = 3c$

Write the equations.

$$\begin{cases} p + a + c = 1000 \\ 2.5p + 6a + 8c = 5000 \\ p - 3c = 0 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1000 \\ 2.5 & 6 & 8 & 5000 \\ 1 & 0 & -3 & 0 \end{array}\right]$$

Rwnd

Write the answers in words, to the nearest tenth of a pound

$\boxed{\begin{array}{l} 352.9 \text{ pounds of peanuts} \\ 529.4 \text{ pounds of almonds} \\ 117.6 \text{ pounds of cashews} \end{array}}$

NO

Part B: Graphing calculator. Regular calculator OK.

3. Consider the matrix

$$A = \begin{bmatrix} 1 & -1 \\ -2 & 6 \end{bmatrix}$$

(a) Write the determinant. Find it.

$$\begin{vmatrix} 1 & -1 \\ -2 & 6 \end{vmatrix} = 1\cdot 6 - (-2)(-1) = 6 - 2 = \boxed{4}$$

(b) Showing the math, find the inverse.

$$A^{-1} = \frac{1}{4}\begin{bmatrix} 6 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix}$$

(c) Use the inverse of the matrix A to solve for the matrix X.

$AX = B$

$$\begin{bmatrix} 1 & -1 \\ -2 & 6 \end{bmatrix} X = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$$

Write the equation for X.

$$X = A^{-1}B = \begin{bmatrix} \frac{3}{2} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix}\begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$$

Write X.

$$X = \begin{bmatrix} \left(\frac{3}{2}\right)(-2) + \left(\frac{1}{4}\right)(1) & \left(\frac{3}{2}\right)(3) + \left(\frac{1}{4}\right)(2) \\ \left(\frac{1}{2}\right)(-2) + \left(\frac{1}{4}\right)(1) & \left(\frac{1}{2}\right)(3) + \left(\frac{1}{4}\right)(2) \end{bmatrix}$$

$$X = \begin{bmatrix} -\frac{11}{4} & 5 \\ -\frac{3}{4} & 2 \end{bmatrix}$$

(d) Solve using the inverse matrix method.

$$\begin{cases} x - y = 5 \\ -2x + 6y = -24 \end{cases}$$

Rewrite the system of equations as a matrix equation.

$$\begin{bmatrix} 1 & -1 \\ -2 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -24 \end{bmatrix}$$

Write the solution equation.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 6 \end{bmatrix}^{-1}\begin{bmatrix} 5 \\ -24 \end{bmatrix}$$

Write x and y.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix}\begin{bmatrix} 5 \\ -24 \end{bmatrix}$$

$$= \begin{bmatrix} \left(\frac{3}{2}\right)(5) + \left(\frac{1}{4}\right)(-24) \\ \left(\frac{1}{2}\right)(5) + \left(\frac{1}{4}\right)(-24) \end{bmatrix} = \begin{bmatrix} \frac{15}{2} - 6 \\ \frac{5}{2} - 6 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} \\ -7/2 \end{bmatrix}$$

Check:

$3/2 - -7/2 = 5$

$-2 \cdot 3/2 + 6 \cdot -7/2 = -24$ ✓

Sheet #452. Systems. IAE 7th Ch 9-3 p. 691

Investment Portfolio **In Exercises 55 and 56, consider an investor with a portfolio totaling $500,000 that is invested in certificates of deposit, municipal bonds, blue-chip stocks, and growth or speculative stocks. How much is invested in each type of investment?**

55. The certificates of deposit pay 10% annually, and the municipal bonds pay 8% annually. ~~Over a five-year period,~~ the investor expects the blue-chip stocks to return 12% annually and the growth stocks to return 13% annually. The investor wants a combined annual return of 10% and also wants to have only one-fourth of the portfolio invested in stocks. Assume $50,000 in growth stocks.

56. The certificates of deposit pay 9% annually, and the municipal bonds pay 5% annually. ~~Over a five-year period,~~ the investor expects the blue-chip stocks to return 12% annually and the growth stocks to return 14% annually. The investor wants a combined annual return of 10% and also wants to have only one-fourth of the portfolio invested in stocks.

Assume $100,000 in growth stocks.
What happens if the investor wants $10,000 in growth stocks?

57. ***Agriculture*** A mixture of 5 pounds of fertilizer A, 13 pounds of fertilizer B, and 4 pounds of fertilizer C provides the optimal nutrients for a plant. Commercial brand X contains equal parts of fertilizer B and fertilizer C. Commercial brand Y contains one part of fertilizer A and two parts of fertilizer B. Commercial brand Z contains two parts of fertilizer A, five parts of fertilizer B, and two parts of fertilizer C. How much of each fertilizer brand is needed to obtain the desired mixture?

Sheet #452. Systems. IAE 7th Ch 10-3 p. 771

There may be a 61 and 62 on a different page.

In Exercises 61–66, use the matrix capabilities of a graphing utility to solve (if possible) the system of linear equations.

61. $\begin{cases} 5x - 3y + 2z = 2 \\ 2x + 2y - 3z = 3 \\ x - 7y + 8z = -4 \end{cases}$

62. $\begin{cases} 2x + 3y + 5z = 4 \\ 3x + 5y + 9z = 7 \\ 5x + 9y + 17z = 13 \end{cases}$

63. $\begin{cases} 3x - 2y + z = -29 \\ -4x + y - 3z = 37 \\ x - 5y + z = -24 \end{cases}$

64. $\begin{cases} -8x + 7y - 10z = -151 \\ 12x + 3y - 5z = 86 \\ 15x - 9y + 2z = 187 \end{cases}$

65. $\begin{cases} 7x - 3y + 2w = 41 \\ -2x + y - w = -13 \\ 4x + z - 2w = 12 \\ -x + y - w = -8 \end{cases}$

66. $\begin{cases} 2x + 5y + w = 11 \\ x + 4y + 2z - 2w = -7 \\ 2x - 2y + 5z + w = 3 \\ x - 3w = -1 \end{cases}$

Sheet #452. Systems. PC 5th Ch. 7-3, p. 586

35. $\begin{cases} 2x + 3y = 0 \\ 4x + 3y - z = 0 \\ 8x + 3y + 3z = 0 \end{cases}$

36. $\begin{cases} 4x + 3y + 17z = 0 \\ 5x + 4y + 22z = 0 \\ 4x + 2y + 19z = 0 \end{cases}$

37. $\begin{cases} 12x + 5y + z = 0 \\ 23x + 4y - z = 0 \end{cases}$

38. $\begin{cases} 2x - y - z = 0 \\ -2x + 6y + 4z = 2 \end{cases}$

In Exercises 39–42, find the equation of the parabola

$y = ax^2 + bx + c$

that passes through the given points. To verify your result, use a graphing utility to plot the points and graph the parabola.

39. $(0, 0), (2, -2), (4, 0)$

40. $(0, 3), (1, 4), (2, 3)$

41. $(2, 0), (3, -1), (4, 0)$

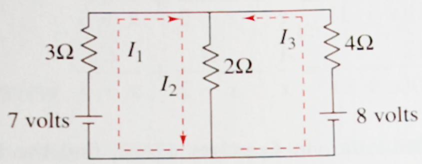
42. $(1, 3), (2, 2), (3, -3)$

Sheet #452. Systems. PC 5th Ch. 7-3, p. 587

60. ***Electrical Network*** Applying Kirchhoff's Laws to the electrical network in the figure, the currents I_1, I_2, and I_3 are the solution of the system

$$\begin{cases} I_1 - I_2 + I_3 = 0 \\ 3I_1 + 2I_2 = 7 \\ 2I_2 + 4I_3 = 8. \end{cases}$$

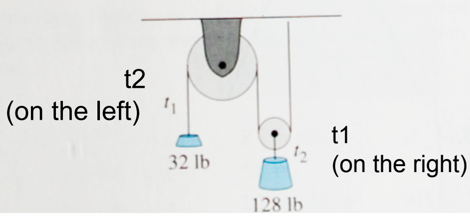
Find the currents.



61. ***Pulley System*** A system of pulleys is loaded with 128-pound and 32-pound weights (see figure). The tensions t_1 and t_2 in the ropes and the acceleration a of the 32-pound weight are found by solving the system of equations

$$\begin{cases} t_1 - 2t_2 = 0 \\ t_1 - 2a = 128 \\ t_2 + a = 32 \end{cases}$$

where t_1 and t_2 are measured in pounds and a is measured in feet per second squared. Solve this system.



62. ***Pulley System*** If the 32-pound weight in the pulley system in Exercise 61 is replaced by a 64-pound weight, the new pulley system will be modeled by the following system of equations.

$$\begin{cases} t_1 - 2t_2 = 0 \\ t_1 - 2a = 128 \\ t_2 + 2a = 64 \end{cases}$$

Solve this system and use your answer for the acceleration to describe what (if anything) is happening in the pulley system.

Sheet#452: Systems. Answers

Page 1

55. \$225,000, \$150,000, \$75,000, \$50,000.
56. \$356,250, \$18,750, \$25,000, \$100,000.
What happens is that it is not possible.
57. 4 pounds, 9 pounds, 9 pounds.

Page 2

61. Infinite number of solutions,
$(5/16a+13/16,\ 19/16a + 11/16,\ a)$
62. Infinite number of solutions,
$(2a-1,\ -3a+2,\ a)$
63. $(-7, 3, -2)$
64. $(10, -3, 5)$
65. $(5, 0, -2, 3)$
66. $(441/71, -163/213, -569/213, 512/213)$

Page 3

35. $(0, 0, 0)$
36. $(0, 0, 0)$
37. $(9/67a, -35/67a, a)$
38. $(1/5a+1/5, -3/5a + 2/5, a)$
39-42. $y = 0.5x^2-2x$, $-x^2+2x+3$, x^2-6x+8, $-2x^2+5x$.

Page 4

60. 1A, 2A, 1A.
61. t_1(right)=96 pounds, t_2(left)=48 pounds, $a= -16$ ft/s^2.
62. t_1(right)=128 pounds, t_2(left)=64 pounds, $a=0$ ft/s^2.

Sheet # 454: Review of Systems with Applications
PC, 5th ed, p. 613.

Solve using an augmented matrix and RREF.

33.

$$\begin{cases} x + 2y + 6z = 4 \\ -3x + 2y - z = -4 \\ 4x + 2z = 16 \end{cases}$$

34.

$$\begin{cases} x + 3y - z = 13 \\ 2x - 5z = 23 \\ 4x - y - 2z = 14 \end{cases}$$

35.

$$\begin{cases} x - 2y + z = -6 \\ 2x - 3y = -7 \\ -x + 3y - 3z = 11 \end{cases}$$

36.

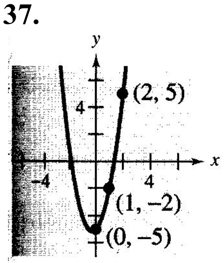
$$\begin{cases} 2x + 6z = -9 \\ 3x - 2y + 11z = -16 \\ 3x - y + 7z = -11 \end{cases}$$

In exercises 37 and 38, find the equation of the parabola

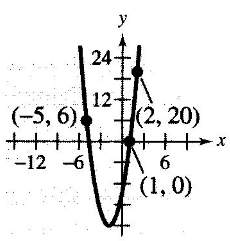
$$y = ax^2 + bx + c$$

that passes through the given points. To verify your result, use a graphing utility to plot the points and graph the parabola.

37.



38.



In Exercises 43 and 44, solve the nonsquare system of equations.

43. $\begin{cases} 5x - 12y + 7z = 16 \\ 3x - 7y + 4z = 9 \end{cases}$

44. $\begin{cases} 2x + 5y - 19z = 34 \\ 3x + 8y - 31z = 54 \end{cases}$

46. ***Finance*** An inheritance of \$40,000 was divided among three investments yielding \$3500 in interest per year. The interest rates for the three investments were 7%, 9%, and 11%. Find the amount placed in each investment if the second and third were \$3000 and \$5000 less than the first, respectively.

45. *Agriculture* A mixture of 6 gallons of chemical A, 8 gallons of chemical B, and 13 gallons of chemical C is required to kill a certain destructive crop insect. Commercial spray X contains 1, 2, and 2 parts, respectively, of these chemicals. Commercial spray Y contains only chemical C. Commercial spray Z contains chemicals A, B, and C in equal amounts. How much of each type of commercial spray is needed to get the desired mixture?

Sheet 454 Answers.

31. $(2, -4, -5)$ **33.** $\left(\frac{24}{5}, \frac{22}{5}, -\frac{8}{5}\right)$

35. $(3a + 4, 2a + 5, a)$ **37.** $y = 2x^2 + x - 5$

39. $x^2 + y^2 - 4x + 4y - 1 = 0$ **41.** $y = 1.01x + 1.54$

43. $(a - 1, a, a + 3)$

45. 10 gallons of spray X

5 gallons of spray Y

12 gallons of spray Z

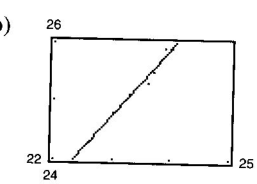
30. $(250{,}000, 95)$ **32.** $(5, -8, 3)$ **34.** $\left(\frac{38}{17}, \frac{40}{17}, -\frac{63}{17}\right)$

36. $\left(-\frac{3}{4}, 0, -\frac{5}{4}\right)$ **38.** $y = 3x^2 + 11x - 14$

40. $x^2 + y^2 - 2x + 2y - 23 = 0$

42. (a) $y = 1.146x - 1.607$

(b)



(c) The line seems to be a good model for the data.

(d) A 1-year change in x results in a 1.146-year change in y.

44. $(-3a + 2, 5a + 6, a)$

46. \$16,000 at 7%

\$13,000 at 9%

\$11,000 at 11%

Sheet 455: Review Matrices

Name ______________________________ Per/Sec. ________________

Always show work. For calculator questions, show the matrices used.

Graphing calculator not allowed

1. $\begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix}$

2. $\begin{vmatrix} -2 & 2 & 3 \\ 5 & 0 & 1 \\ 6 & -4 & -4 \end{vmatrix}$

3. Solve using Cramer's Method. $x + 6y = -9$
   $x - 3y = 6$

4. Solve using the inverse matrix. $-3a + b = -3$
   $5a - 2b = 10$

Graphing calculator allowed

5. $4x + y = -7$
   $x - 2z = 4$
   $3y + 2z = -3$

6. A local theater charges for $2.25 for general admission and $3.50 for box office seats. For a certain performance, 165 tickets were sold, which brought in a total of $482.50. Find out how many of each type of ticket were sold.

7. From two kinds of corn meal, one costing $0.65/kg and the other $0.85/kg, a cattle farmer wishes to make a 100 kilogram mixture costing $0.76/kg. How many kilograms of each kind should be used?

8. An airline maintains three different classes of service in a Boeing 747: first, business, and economy. The configuration of the plane (number of seats in each class) is based on demand. There are 42 seats available for either first or business class, and 570 seats for either business or economy class. If, on a certain flight, there are 34 times as many economy as first class seats, what is the configuration of the plane?

Answer

1. 5
2. -16
3. $(1, -\frac{5}{3})$
4. $(-4, -15)$
5. $(-2, 1, -3)$

76 general, 89 box

55kg at $0.85, 45kg at $0.65

16 first class, 26 business, 544 economy

KEY

Sheet 455: Review Matrices

Name ____KEY____ Per/Sec. ______________

Always show work. For calculator questions, show the matrices used.

Graphing calculator not allowed

1. $\begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix} = 4\cdot 2 - (3\cdot 1) = \boxed{5}$

2. $\begin{vmatrix} -2 & 2 & 3 \\ 5 & 0 & 1 \\ 6 & -4 & -4 \end{vmatrix} = 0 + 2\cdot 1\cdot 6 + 5(-4)(3) - 0 - 5(2)(-4) - (-2)(-4)(1) = 12 - 60 + 40 - 8 = \boxed{-16}$

3. Solve using Cramer's Method. $x + 6y = -9$, $x - 3y = 6$

$A = \begin{bmatrix} 1 & 6 \\ 1 & -3 \end{bmatrix}$. $|A| = (1)(-3) - (1)(6) = -9$

$x = \dfrac{\begin{vmatrix} -9 & 6 \\ 6 & -3 \end{vmatrix}}{-9} = \dfrac{27-36}{-9} = \dfrac{-9}{-9} = 1$

$y = \dfrac{\begin{vmatrix} 1 & -9 \\ 1 & 6 \end{vmatrix}}{-9} = \dfrac{6+9}{-9} = \dfrac{-15}{9} = \dfrac{-5}{3}$

$\boxed{(x, y) = (1, \tfrac{-5}{3})}$

4. Solve using the inverse matrix. $-3a + b = -3$, $5a - 2b = 10$

$\begin{bmatrix} -3 & 1 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -3 \\ 10 \end{bmatrix}$, $A^{-1} = \dfrac{1}{|A|}\begin{bmatrix} -2 & -1 \\ -5 & -3 \end{bmatrix}$

$\begin{vmatrix} -3 & 1 \\ 5 & -2 \end{vmatrix} = 6 - 5 = 1$

$\begin{bmatrix} a \\ b \end{bmatrix} = \dfrac{1}{1}\begin{bmatrix} -2 & -1 \\ -5 & -3 \end{bmatrix}\begin{bmatrix} -3 \\ 10 \end{bmatrix} = \begin{bmatrix} -2\cdot(-3) - 1(10) \\ -5(-3) - 3(10) \end{bmatrix} = \begin{bmatrix} -4 \\ -15 \end{bmatrix}$

$\boxed{(a, b) = (-4, -15)}$

Graphing calculator allowed

5. $4x + y = -7$
$x - 2z = 4$
$3y + 2z = -3$

rref of $\left[\begin{array}{ccc:c} 4 & 1 & 0 & -7 \\ 1 & 0 & -2 & 4 \\ 0 & 3 & 2 & -3 \end{array}\right]$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$ $\boxed{(x, y, z) = (-2, 1, -3)}$

6. A local theater charges for \$2.25 for general admission and \$3.50 for box office seats. For a certain performance, 165 tickets were sold, which brought in a total of \$482.50. Find out how many of each type of ticket were sold.

$\begin{cases} 2.25g + 3.50\cdot b = 482.50 \\ g + b = 165 \end{cases}$

$b = 165 - g$ (I picked substitution method)

$2.25g + 3.5(165 - g) = 482.5$

$95 = 1.25g \to g = 76$

$\boxed{\text{76 general, 89 box office}}$

7. From two kinds of corn meal, one (a) costing \$0.65/kg and the other (b) \$0.85/kg, a cattle farmer wishes to make a 100 kilogram mixture costing \$0.76/kg. How many kilograms of each kind should be used?

$\begin{cases} a + b = 100 \\ 0.65a + 0.85b = 0.76\cdot 100 \end{cases}$

$\begin{bmatrix} 1 & 1 \\ 0.65 & 0.85 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 100 \\ 76 \end{bmatrix}$

rref of $\left[\begin{array}{cc:c} 1 & 1 & 100 \\ 0.65 & 0.85 & 76 \end{array}\right]$

Corn meal 1 : 45 kg at 0.65/kg
Corn meal 2 : 55 kg at \$0.85/kg.

8. An airline maintains three different classes of service in a Boeing 747: first, business, and economy. The configuration of the plane (number of seats in each class) is based on demand. There are 42 seats available for either first or business class, and 570 seats for either business or economy class. If, on a certain flight, there are 34 times as many economy as first class seats, what is the configuration of the plane?

$\begin{cases} f + b = 42 \\ b + e = 570 \\ 34f - e = 0 \end{cases}$

rref of $\left[\begin{array}{ccc:c} 1 & 1 & 0 & 42 \\ 0 & 1 & 1 & 570 \\ 34 & 0 & -1 & 0 \end{array}\right]$

first class = 16 seats
business class : 26 seats
economy class : 544 seats

checks :
16 + 26 = 42
26 + 544 = 570
6·34 = 544!

Answer

76 general, 89 box
55kg at \$0.85, 45kg at \$0.65
16 first class, 26 business, 544 economy

1. 5
2. −16
3. $(1, -\frac{5}{3})$
4. $(-4, -15)$
5. $(-2, 1, -3)$

Name: ___________________

Sheet 456: Three-Dimensional System Word Problems

Instructions: Write equations and matrix equation. Solve in any fashion. Reply in complete sentences.

1. Billy's Restaurant ordered 200 flowers for Mother's Day. They ordered carnations at $1.50 each, roses at $5.75 each, and daisies at $2.60 each. They ordered mostly carnations, and 20 fewer roses than daisies. The total order came to $589.50. How many of each type of flower was ordered?

2. Last Tuesday, Regal Cinemas sold a total of 8500 movie tickets. Proceeds totaled $64,600. Tickets can be bought in one of 3 ways: a matinee admission costs $5, student admission is $6 all day, and regular admissions are $8.50. How many of each type of ticket was sold if twice as many student tickets were sold as matinee tickets?

80 50 70; 900 1800 5800

Sheet 457: Interest rate problem: system with three unknowns

TeeMS Company borrowed $100,000 to purchase a T-shirt printing press. The money was borrowed at annual interest rates of 5%, 7%, and 11%, respectively. The annual interest owed was $6,500. The amount borrowed at 7% was four times the amount of the amount borrowed at 11%. How much money was borrowed at each interest rate?

a) Write the equations. What do the variables stand for?

b) Write the augmented matrix (or the matrix equation).

c) Solve. Given answers in words and numbers.