

Determinants of 2x2, 3x3 and 4x4 square matrices

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} + & - \\ - & + \end{pmatrix}$$

$$A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$$

$$| \begin{smallmatrix} 2 & 3 \\ 4 & 5 \end{smallmatrix} | = \det A = (2)(5) - (4)(3) = 10 - 12 = -2.$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$$

Expansion about Row 1.

$$A = \begin{pmatrix} 0 & 2 & 1 \\ 3 & -1 & 2 \\ 4 & 0 & 1 \end{pmatrix}$$

$$\det A = +0 \cdot \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} - 2 \cdot \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix} + 1 \cdot \begin{vmatrix} 3 & -1 \\ 4 & 0 \end{vmatrix}$$

$$= 0 - 2(3-8) + (0+4) = 0 + 10 + 4 = 14.$$

Expansion about Column 2.

$$\det A = -2 \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix} + (-1) \begin{vmatrix} 0 & 1 \\ 3 & 2 \end{vmatrix} - 0 \begin{vmatrix} 0 & 1 \\ 3 & 2 \end{vmatrix}$$

$$= -2(3-8) - (0-4) - 0 = +10 + 4 = 14 \checkmark$$

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ -3 & -2 & 0 \end{pmatrix}$$

Top Row $\det A = 1 \cdot (0+2) - 2(0+3) + (-1)(-6-0)$

$$= 2 - 6 + 6 = 2.$$

$$1 \begin{vmatrix} 0 & 1 \\ -2 & 0 \end{vmatrix} - 2 \begin{vmatrix} 3 & 1 \\ -3 & 0 \end{vmatrix} + (-1) \begin{vmatrix} 3 & 0 \\ -3 & -2 \end{vmatrix}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \begin{pmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & -2 & 3 & 0 \\ -1 & 1 & 0 & 2 \\ 0 & 2 & 0 & 3 \\ 3 & 4 & 0 & 2 \end{pmatrix}$$

 $\det A =$

EXPAND COLUMN 3.

$$3 \begin{vmatrix} -1 & 1 & 2 \\ 0 & 2 & 3 \\ 3 & 4 & 2 \end{vmatrix} + 0 + 0 + 0 = 3 \cdot 5 = \boxed{15}$$

$$\begin{vmatrix} -1 & 1 & 2 \\ 0 & 2 & 3 \\ 3 & 4 & 2 \end{vmatrix} = -0 + 2 \begin{vmatrix} -1 & 2 \\ 3 & 2 \end{vmatrix} - 3 \begin{vmatrix} -1 & 1 \\ 3 & 4 \end{vmatrix} = -2(-2-6) - 3(-4-3)$$

$$= 2(-8) - 3(-7) = -16 + 21 = 5$$