Name: KEY

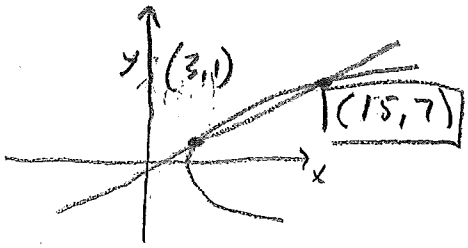
Sheet 327: Linear and Quadratic Systems Review 2

For each problem, do all of these:

A. Write down each of the "$y =$" equations.
B. Sketch
C. Find all intersections (if any).

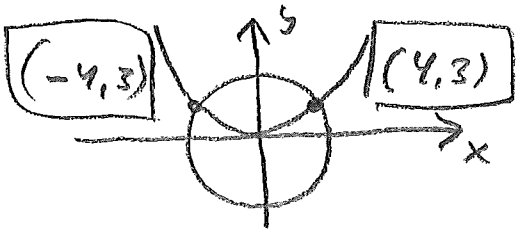
1. $\begin{cases} y^2 - 4x + 11 = 0 \\ -\frac{1}{2}x + y = -\frac{1}{2} \end{cases}$

$\begin{cases} y_1 = \sqrt{4x-11} \\ y_2 = -\sqrt{4x+11} \\ y_3 = \frac{1}{2}x - \frac{1}{2} \end{cases}$



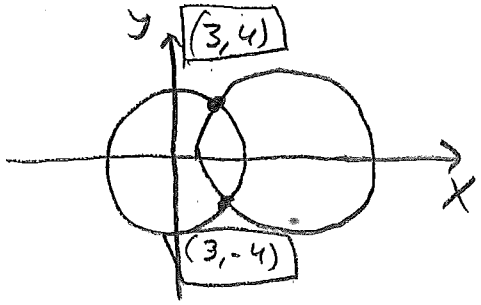
2. $\begin{cases} x^2 + y^2 = 25 \\ 3x^2 - 16y = 0 \end{cases}$

$\begin{cases} y_1 = \sqrt{25-x^2} \\ y_2 = -\sqrt{25-x^2} \\ y_3 = 3x^2/16 \end{cases}$



3. $\begin{cases} x^2 + y^2 = 25 \\ (x-8)^2 + y^2 = 41 \end{cases}$

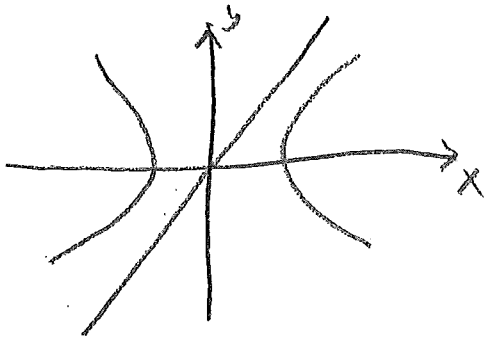
$\begin{cases} y_1 = \sqrt{25-x^2} \\ y_2 = -\sqrt{25-x^2} \\ y_3 = \sqrt{41-(x-8)^2} \\ y_4 = -\sqrt{41-(x-8)^2} \end{cases}$



4. $\begin{cases} 3x - 2y = 0 \\ x^2 - y^2 = 4 \end{cases}$

$\begin{cases} y_1 = 3/2\,x \\ y_2 = \sqrt{x^2-4} \\ y_3 = -\sqrt{x^2-4} \end{cases}$

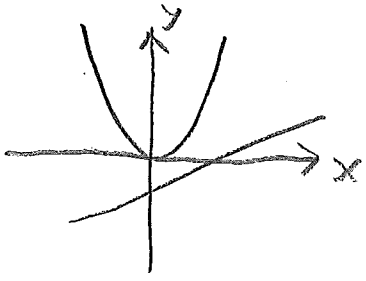
NO INTERSECTION



5. $\begin{cases} x - 2y = 4 \\ x^2 - y = 0 \end{cases}$

$\begin{cases} y = \frac{1}{2}x - 2 \\ y = x^2 \end{cases}$

NO INTERSECTION



Name KEY

Sheet 455: Review Matrices

Per/Sec. _____

Always show work. For calculator questions, show the matrices used.

Graphing calculator not allowed

1. $\begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix} = 4\cdot 2 - (3\cdot 1) = \boxed{5}$

2. $\begin{vmatrix} -2 & 2 & 3 \\ 5 & 0 & 1 \\ 6 & -4 & -4 \end{vmatrix} = 0 + 2\cdot 1\cdot 6 + 5(-4)(3) - 0 - 5(2)(-4) - (-2)(-4)(1)$
$= 12 - 60 + 40 - 8 = \boxed{-16}$

3. Solve using Cramer's Method. $x + 6y = -9$, $x - 3y = 6$

$A = \begin{bmatrix} 1 & 6 \\ 1 & -3 \end{bmatrix}$. $|A| = (1)(-3) - (1)(6) = -9$

$x = \dfrac{\begin{vmatrix} -9 & 6 \\ 6 & -3 \end{vmatrix}}{-9} = \dfrac{27-36}{-9} = \dfrac{-9}{-9} = 1$

$y = \dfrac{\begin{vmatrix} 1 & -9 \\ 1 & 6 \end{vmatrix}}{-9} = \dfrac{6+9}{-9} = \dfrac{15}{-9} = \dfrac{-5}{3}$

Graphing calculator allowed $\boxed{(x,y) = (1, \frac{-5}{3})}$

4. Solve using the inverse matrix. $-3a + b = -3$, $5a - 2b = 10$

$\begin{bmatrix} -3 & 1 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -3 \\ 10 \end{bmatrix}$, $A^{-1} = \dfrac{1}{|A|}\begin{bmatrix} -2 & -1 \\ -5 & -3 \end{bmatrix}$

$\begin{vmatrix} -3 & 1 \\ 5 & -2 \end{vmatrix} = 6 - 5 = 1$

$\begin{bmatrix} a \\ b \end{bmatrix} = \dfrac{1}{1}\begin{bmatrix} -2 & -1 \\ -5 & -3 \end{bmatrix}\begin{bmatrix} -3 \\ 10 \end{bmatrix} = \begin{bmatrix} -2(-3) - 1(10) \\ -5(-3) - 3(10) \end{bmatrix} = \begin{bmatrix} -4 \\ -15 \end{bmatrix}$

$\boxed{(a,b) = (-4, -15)}$

5. $4x + y = -7$
$x - 2z = 4$
$3y + 2z = -3$

rref of $\begin{bmatrix} 4 & 1 & 0 & -7 \\ 1 & 0 & -2 & 4 \\ 0 & 3 & 2 & -3 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$ $\boxed{(x,y,z) = (-2, 1, -3)}$

6. A local theater charges for \$2.25 for general admission and \$3.50 for box office seats. For a certain performance, 165 tickets were sold, which brought in a total of \$482.50. Find out how many of each type of ticket were sold.

$\begin{cases} 2.25g + 3.50\cdot b = 482.50 \\ g + b = 165 \end{cases}$

$b = 165 - g$ (I picked substitution method)

$2.25g + 3.5(165 - g) = 482.5$

$95 = 1.25g \rightarrow g = 76$

$\boxed{76 \text{ general}, 89 \text{ box office}}$

7. From two kinds of corn meal, a one costing \$0.65/kg and the b other \$0.85/kg, a cattle farmer wishes to make a 100 kilogram mixture costing \$0.76/kg. How many kilograms of each kind should be used?

$\begin{cases} a + b = 100 \\ 0.65a + 0.85b = 0.76\cdot 100 \end{cases}$

$\begin{bmatrix} 1 & 1 \\ 0.65 & 0.85 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 100 \\ 76 \end{bmatrix}$

rref of $\begin{bmatrix} 1 & 1 & 100 \\ 0.65 & 0.85 & 76 \end{bmatrix}$

$\boxed{\begin{array}{l} \text{Cornmeal 1} = 45 \text{ kg at } 0.65/\text{kg} \\ \text{Cornmeal 2} = 55 \text{ kg at } \$0.85/\text{kg} \end{array}}$

8. An airline maintains three different classes of service in a Boeing 747: first, business, and economy. The configuration of the plane (number of seats in each class) is based on demand. There are 42 seats available for either first or business class, and 570 seats for either business or economy class. If, on a certain flight, there are 34 times as many economy as first class seats, what is the configuration of the plane?

$\begin{cases} f + b = 42 \\ b + e = 570 \\ 34f - e = 0 \end{cases}$

rref of $\begin{bmatrix} 1 & 1 & 0 & 42 \\ 0 & 1 & 1 & 570 \\ 34 & 0 & -1 & 0 \end{bmatrix}$

$\boxed{\begin{array}{l} \text{first class} = 16 \text{ seats} \\ \text{business class} = 26 \text{ seats} \\ \text{economy class} = 544 \text{ seats} \end{array}}$

checks =
$16 + 26 = 42$
$26 + 544 = 570$
$16 \cdot 34 = 544$

Answer

1. 5
2. −16
3. $(1, -\frac{5}{3})$
4. $(-4, -15)$
5. $(-2, 1, -3)$

76 general, 89 box
55 kg at \$0.85, 45 kg at \$0.65
16 first class, 26 business, 544 economy

Sheet 1141: Sequences and Series Review

Sheet#1141: Seq Ser Review

Name KEY

Per/Sec. _____

State the first 5 terms of the sequence whose nth term is given by a_n.

1. $a_n = -\frac{3}{n}$

$a_1 = -\frac{3}{1} = \boxed{-3}$

$a_2 = \boxed{-\frac{3}{2}}$

$a_3 = -\frac{3}{3} = \boxed{-1}$

$a_4 = \boxed{-\frac{3}{4}}$

$a_5 = \boxed{-3/5}$

State the next 2 terms of the sequence and give a formula for the nth term.

2. 63, 54, 45, 36, 27, 18, 9.

-9 -9

$a_n = 63 + (n-1)\cdot(-9)$

$a_n = 63 - 9n + 9$ $\boxed{a_n = 72 - 9n}$

Find the sum.

3. $\sum_{n=1}^{4} n^4 = 1^4 + 2^4 + 3^4 + 4^4 =$

$= 1 + 16 + 81 + 256 = \boxed{354}$

Express using sigma notation.

4. $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \frac{1}{81} - \frac{1}{243}$

First term = 1.

$\boxed{\sum_{i=1}^{6} \left(\frac{-1}{3}\right)^{i-1}} = \sum_{i=0}^{5} \left(\frac{-1}{3}\right)^{i}$

5. State the 8th term of the geometric sequence $-9, \frac{9}{2}, -\frac{9}{4}, \ldots$.

$a_n = -9\left(-\frac{1}{2}\right)^{n-1}$

$a_8 = -9\left(-\frac{1}{2}\right)^{7} = \boxed{\frac{9}{128}}$

6. The first 3 terms of an arithmetic progression are -15, -17, and -19. Find the 22nd term and the sum of the first 22 terms.

$d = -2$

$a_n = -15 + (n-1)\cdot(-2)$

$a_{22} = -15 + (21)(-2) = \boxed{-57}$

$S_{22} = \frac{1}{2}(-15 - 57)\cdot 22$

$= \boxed{-792}$

7. How many terms of the arithmetic series $(-15) + (-8) + (-1) + \cdots$ must be added to give a sum of 801?

ANSWER: $\boxed{18 \text{ terms}}$

$d = 7$

$a_n = -15 + (n-1)\cdot 7$

$a_n = -15 + 7n - 7$

$a_n = -22 + 7n$

$801 = \frac{1}{2}\left(\underbrace{-15 + [-22 + 7n]}_{-37+7n}\right)\cdot n$

$801 = \frac{1}{2}(-37 + 7n)\cdot n$

$1602 = -37n + 7n^2$ QUADRATIC EQUATION

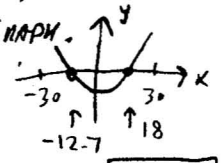
$7n^2 - 37n - 1602 = 0$

SOLVE BY QUADRATIC FORMULA OR GRAPH.

$x = \frac{37 \pm \sqrt{37^2 + 4\cdot 7\cdot 1602}}{(2\cdot 7)}$

$x = \frac{37 \pm \sqrt{46225}}{14}$

$x = \frac{37 \pm 215}{7} = \begin{cases} 18 \\ -89/7 \approx -12.7 \end{cases}$



$\boxed{N = 18}$

8. Find the sum of the first 6 terms of the geometric series $80 + (-20) + 5 + \cdots$.

$r = -1/4$

$S_6 = 80\left(\frac{1 - (-1/4)^6}{1 - (-1/4)}\right) = 80\left(\frac{1 - \frac{1}{4096}}{1 + 1/4}\right)$

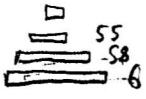
$= 80\,\frac{\left(\frac{4095}{4096}\right)}{\left(\frac{5}{4}\right)} = \frac{80\cdot 4095\cdot 4}{4096\cdot 5} = \boxed{\frac{4095}{64}}$

9. $\frac{1}{2} + \left(-\frac{1}{4}\right) + \frac{1}{8} + \cdots$

$r = \frac{-1}{2}$ $a = \frac{1}{2}$

$S = \frac{1}{2}\left(\frac{1}{1 - (-1/2)}\right) = \frac{1}{2}\left(\frac{1}{3/2}\right) = \boxed{\frac{1}{3}}$

10. A log pile has 61 logs in the bottom layer, 58 in the second layer, 55 in the third layer, and so on. If there are 639 logs in the pile, how many layers are there?



639 logs.

METHOD 2: Add until you get to 639.

$61 + 58 + 55 + \cdots + 10 = 639$.

LAYERS = 18

METHOD 1: $639 = \frac{1}{2}\left(61 + [61 + (N-1)(-3)]\right)\cdot N$

$1278 = (122 - 3N + 3)N$

$3N^2 - 125N + 1278 = 0$

SOLVE: $N = 23.67$ or $N = 18$

$\boxed{18 \text{ LAYERS}}$

11. After the first swing, the path of a pendulum bob is 0.9 as long as long as the previous swing. If the first swing is 50 cm long, how far does the bob travel on the sixth swing? What total distance does the bob travel in those six swings?

$a_n = 50(0.9)^{n-1}$

$a_6 = 50\cdot(0.9)^5 \approx \boxed{29.5 \text{ cm}}$

$S_6 = 50\left(\frac{1 - 0.9^6}{1 - 0.9}\right) = \boxed{234.3 \text{ cm}}$

12. **Formulas**

- Arithmetic sequences and series:

$a_n = a_1 + (n-1)d$ and $S_n = n\left(\frac{a_1 + a_n}{2}\right)$.

- Geometric sequences and series:

$a_n = a_1(r)^{n-1}$ and $S_n = a_1\left(\frac{1 - r^n}{1 - r}\right)$, and

for $n \to \infty$ with $|r| < 1$, $S = a_1\left(\frac{1}{1 - r}\right)$.